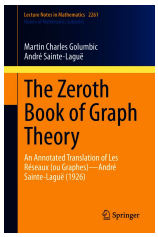
Review of [1]

The Zeroth Book of Graph Theory: An Annotated Translation of Les Réseaux (ou Graphes) – André Sainte-Laguë (1926)

by Martin Charles Golumbic and André Sainte-Laguë



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1 Overview

The Zeroth Book of Graph Theory is the first and only English translation of André Sainte-Laguë's 1926 monograph *Les Réseaux (ou Graphes)*. Preceding the first full book on graph theory by a decade, Sainte-Laguë's monograph is a record of the state of graph theory research in its earliest days. The book begins with the basic definitions commonly known at the time and then moves on to a variety of graph-theoretic and combinatorial topics.

Martin Golumbic's translation maintains the illustrations, section numbers, and formatting of the original French version. It also preserves the mathematical notation used by Sainte-Laguë. The annotations include historical notes, modern equivalents of Sainte-Laguë's terminology, and corrections to mathematical and typographical errors. A glossary of French terms and their English translations chosen by Golumbic is provided along with a short biography of Sainte-Laguë and an extensive bibliography.

2 Summary of Contents

The following chapter summaries pertain to the chapters in *Les Réseaux (ou Graphes)*. I have also added a summary of the commentary provided by the translator/annotator to each chapter.

Chapter 1. Introduction and definitions. Sainte-Laguë motivates the monograph as a study addressing the topological problem of the possibility of certain relative placements of objects and the number of ways in which that placement can be made. He warns the reader against thinking that graph theory applies only to "curiosities" like the Königsberg Bridges and Four-Colour problems. The chapter proceeds with the standard definitions of beginning graph theory.

Golumbic's commentary situates Sainte-Laguë's work in its historical context. He reminds us that at this time most work in combinatorics was considered recreational mathematics.

Chapter 2. Trees. Sainte-Laguë investigates ways of characterizing trees and the use of their centers to count the number of non-isomorphic trees with n internal nodes.

Golumbic's commentary highlights Sainte-Laguë's use of pictures in his lectures at the Conservatoire National des Arts et Métiers.

Chapter 3. Chains and cycles. Sainte-Laguë discusses Eulerian cycles and paths and some combinatoric problems related to these. He also introduces Hamiltonian cycles. He cites Lucas' observation that a path through a labyrinth can be found by viewing the labyrinth as a graph and using depth-first search.

Golumbic's commentary addresses the Königsberg Bridges Problem and Lucas' work with mazes and recreational mathematics.

Chapter 4. Regular graphs. Sainte-Laguë studies a variety of topics related to regular graphs (all vertices have the same degree) with a focus on polygonal graphs. One question of interest is "Given a circle divided into equal parts, what is the number of distinct convex polygons with vertices among the division points?"

Golumbic's commentary presents the Petersen graph alluded to by Sainte-Laguë.

Chapter 5. Cubic graphs. Cubic graphs are regular graphs in which the vertices have degree three. In this chapter Sainte-Laguë investigates bipartite cubic graphs as a gateway to the study of the Four-Colour Problem.

Golumbic does not offer commentary in this chapter, but a footnote by Myriam Preissman situates Sainte-Laguë's work in the context of contemporary knowledge of the Four-Colour problem.

Chapter 6. Tableaux. Sainte-Laguë introduces incidence matrices and some graph properties obtainable from these matrices. His eye is on the result that "the edge-chromatic number (the minimal number of groups into which we can divide all edges so that two edges in the same group never have a common vertex) of a bipartite graph is equal to the maximum of its degrees."

Preissman provided the commentary for this chapter explaining Sainte-Laguë's argument and correcting errors/typos in the original text.

Chapter 7. Hamiltonian graphs. Sainte-Laguë associates permutations with regular Hamiltonian graphs and then classifies the permutations according to the properties of the corresponding graph. This leads to a discussion of the postage stamp problem: In how many ways can one fold a strip of postage stamps?

Golumbic's commentaries (1) suggest that Sainte-Laguë had seen the set of four stamps commemorating the 1924 Olympics and (2) point out the popularity of the mathematical analysis of fabrics, mosaics, and chessboards among 19th century mathematicians.

Chapter 8. Chessboard problems. Sainte-Laguë addresses the well-known problem of placing n chess pieces in non-attacking positions on a chessboard. His investigation includes rooks, queens, and knights. He also looks at variations of these problems, for example, placing n queens that attack exactly two others.

Golumbic's commentary provides a diagram for one of Sainte-Laguë's examples and an etymological remark on the French phrase *en prise* ("can be taken").

Chapter 9. Knight's tour. Sainte-Laguë looks at a variety of solutions to the Knight's Tour problem including those of Euler, Vandermonde, Bertrand, deMoivre, and Roget. The reader will find these different approaches a nice addition to the standard solution seen in modern texts. Sainte-Laguë finishes the chapter with a discussion of a relationship between the Knight's Tour solution and magic squares.

Golumbic's commentary informs us that the Roget mentioned by Sainte-Laguë is the same Peter Mark Roget of *Roget's Thesaurus* fame.

Chapter 10. Conclusion. This extremely short (two paragraphs) chapter sums up Sainte-Laguë's view that the study of graphs "can be pursued in many different ways, and in each of the

notions defined may initiate new research" and "as limited as it may appear at first, is in fact vast and seems quite difficult." There is no commentary on this chapter.

The Zeroth Book of Graph Theory concludes with short biographies of André Sainte-Laguë and of Guy Ghidale Iliovici. (Iliovici was a friend and colleague of Sainte-Laguë's who lost his job in occupied France during WWII because he was Jewish. Iliovici was murdered by the Nazis at Auschwitz in 1942.)

The biographies are followed by a bibliography and a glossary of the French terms used by Sainte-Laguë and Golumbic's translations of them. The bibliography includes 223 works cited by Sainte-Laguë and an additional 80 provided by Golumbic. (I was surprised that no works by Frank Harary appeared in Golumbic's list.)

3 Opinion

Any assessment of *The Zeroth Book of Graph Theory* must be a tripartite discussion addressing the content of *Les Réseaux (ou Graphes)* itself, the English translation of the 1926 work, and the commentary and explanatory matter accompanying the translation.

a. *Les Réseaux (ou Graphes)* is an interesting look into the early development of graph theory. The reader will find many familiar themes in Sainte-Laguë's work. These include counting non-isomorphic graphs with specified properties, enumerating paths through graphs, and the popular chessboard problems. It was particularly interesting to see these early struggles with the Four-Color Problem.

   The style of presentation in 1926 was quite a bit different from what we are accustomed to in mathematical works. Theorems and proofs are not marked as such nor set off from the main text. Instead, results blend into the general narration. Any reader interested in the history of graph theory or looking for novel approaches to chessboard problems or the Knight's Tour will find items of interest in Sainte-Laguë's book.

b. I cannot comment of the quality of Golumbic's translation because my command of French is not at the level of a translator. However, the glossary of French terms is quite extensive and would be useful for a scholar reading Sainte-Laguë in the original. I was able to follow along at a rudimentary level to get a feel for Sainte-Laguë's presentation and the choices Golumbic made in his translation. For those who read French, the original is available at `http://archive.numdam.org/item/MSM_1926__18__1_0.pdf`.

c. *The Zeroth Book of Graph Theory* is extensively footnoted. These include modern references as well as historical remarks provided by Robin Wilson, and technical remarks provided by Myriam Preissman and Alan Hertz. Corrections of mathematical and typographical errors were immensely helpful in reading the text. The footnotes include those of *Les Réseaux (ou Graphes)* as well as of *The Zeroth Book of Graph Theory*. When Sainte-Laguë has provided the footnote in the original monograph, Golumbic provides links to the work if it is currently available. This is an excellent feature because it allows the reader to see what Sainte-Laguë saw and inspired him to include it in his work.

   In general, the commentaries were informative. They connected the topics addressed by Sainte-Laguë to modern results, to historical context, and in some cases, to popular culture (e.g., *Star Wars*).

Although not a work from which one should begin one's study of graph theory, I can recommend *The Zeroth Book of Graph Theory* to anyone with an interest in the history of graph theory, the recreational aspects of graph theory, or the evolution of the style of mathematical exposition.