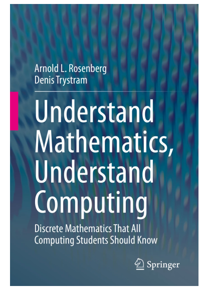
Review of [1]



Understand Mathematics, Understand Computing
Arnold L. Rosenberg and Denis Trystram
Springer, 2020
USD 73.61, Hardcover (also available in Softcover), 577 pages

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1 Overview

This book, subtitled *Discrete Mathematics That All Computing Students Should Know*, is, in addition to being a textbook for an introductory undergraduate course on discrete mathematics, an enthused, extensive, wide-ranging, and detailed overview of and introduction to the discrete mathematics that, well, all of us, not just computing students, really ought to know. My SB('76) and MS('84) in Comp. Sci. were from a distant past, and thus this book is exactly the review and overview I was looking for. However, while the book provided what I needed for the things that I missed or hadn't formally studied (in particular, SAT, graph theory, and statistics and probability), for the material that I was reasonably aware of, i.e., basic number theory, proof, logic, and sets, I found myself muttering "Huh? What are you talking about?" and running to other, more detailed, more technical, or more standard sources far more often than I should have had to.

Despite my kvetching (and most of this review will be kvetching), this book covers a lot of math in a compact package; it's the smallest and lightest of the competing textbooks at hand. It can be read on a favorite reading chair or sofa; the competing texts all require clearing space on a desk. Even the older ones from the 1990s.

Who is this book for? Although I have my doubts about this book as a textbook (it needs major editing of the writing style and careful rewriting to provide all the prerequisites and definitions a non-expert reader would need), it's definitely worth a read for someone, say, about to teach a course on this material or someone interested in reviewing the material with the intention of looking into it more deeply.

What's in the book? A true wealth of computation-relevant mathematics. The first two chapters are introductory material[2], followed by eleven chapters with subject-specific content[3] and seven similarly content-dense appendices. And the authors make a point of providing a proof (and often multiple proofs) when possible. Sure, the usual suspects (sets, logic, numbers, infinities, recurrence

[2] Chapter 1 is the usual bookkeeping for organizing a course, and Chapter 2 is an introduction to proof techniques.
[3] From sets and logic (Chapter 3), through several chapters on numbers and numbers systems, to chapters on combinatorics and graphs.

relations, counting and combinatorics, graphs) all make major appearances, but fun stuff, such as a proof of Fermat's little theorem, some of the joys of the Fibonacci numbers, and the clearest explanation of the Monty Hall problem I've ever read are here as well.

What I think this book is and what the authors think it is, differ. The authors' stated target audience is undergraduates from multiple fields who have not seen this material before. I'm not convinced. The main problem is that the authors often jump ahead of themselves into the content before providing the definitions and explanations needed. The authors' intention, perhaps, was to not write a boring, standard, ordinary textbook, and in avoiding that, they left out that which is good about ordinary textbooks: examples and orderly, thorough presentations. (Again, in reading this book, I found I needed to look for better definitions and descriptions of things in other textbooks far more often than I should have had to.)

But the authors disagree with me and argue strongly that their presentation of this material will enable students to actually do mathematics. Their claims are explicated in the "Manifesto" and Preface, which can be read in the preview on Amazon.

A word about the exercises. The exercises are more extensions of the text than the calculation exercises one would find in, say, a calculus text. While they are similar to those in other discrete mathematics texts, they extend the content of their chapters well. Solutions are provided for the most difficult exercises, and hints for the harder of the other exercises. There are not an excessive number of exercises, but they include interesting material. Problem 2 in Chapter 4 asks the reader to show how to compute the product of two complex numbers using only three multiplications *without any hints or solution*, thus indicating that the authors see it as an easy problem. (I remembered that there was a trick but was unable to remember or reinvent it, to my chagrin.) A problem in a later chapter revisits this idea (in the context of the multiplication of large integers) and does provide an answer. The Josephus problem is another such exercise.

2 How the Material is Presented

This book focuses, quite sensibly, on proofs. It also make a point of providing multiple proofs as often as possible, which is one of the things that attracted me to the book in the first place. However, "proof" here means algebraic proof from basic principles; no higher math. For a sophomore level textbook, this might seem a reasonable approach, but it has its problems. For starters, much of this material is number theory, yet number theory makes no appearance in either the table of contents nor in the index (although there are at least two occurrences in the text). My feeling here is that a short introduction to number theory would have made some of this material easier to present and understand. Similarly for abstract algebra: The book uses some fairly sophisticated concepts (e.g., free algebras and semigroups) with far from adequate explanation, if any.

In actual practice, this works for the majority of the material. But there are cases where this struck me as problematic, such as the discussion of Boolean algebras, which I mention below.

I noticed two proofs which were, for me, "incomplete". What I mean by that is that the last line prior to the QED was one algebraic manipulation short of getting back to the thing to be proved. For example, to show that the sum of the integers from 1 to m is $(m)(m+1)/2$, the last line in the proof as shown (from adding $(m+1)$ to both $1 \ldots m$ and the formula) is $(m+1)(m+2)/2$. But that needs one more step to get to $(m+1)((m+1)+1)/2$, i.e., the exact form of the formula being proved with $(m+1)$ replacing m.

This may sound like a quibble, but I don't think it is. In Michael Penn's YouTube videos [4], he is very careful to clearly show that he actually has gotten back to the thing to be proved. Maybe leaving out the final step as obvious is normal in published papers, but it's not the right thing for a textbook. A sophomore level undergraduate text needs to be more user-friendly.

Another issue which I think is important is that there are cases where the authors either don't define their notation before use or define it in the text in passing (i.e., not clearly marked as important), and that notation then becomes critical for a following discussion. One example of this is that $\mathbb{N}^+$ is used on p. 37 but is not defined until p. 106 and doesn't appear in the list of symbols at the back of the book.

Aside: In computer science, we find it natural to include zero in the natural numbers. So the authors' use of $\mathbb{N}$ and $\mathbb{N}^+$ to express the natural numbers including zero and the positive natural numbers is, of course, quite natural for computer science. But it is common to the point of being nearly universal in higher (that is, upper level undergraduate) mathematics texts to use $\mathbb{N}$ for the positive natural numbers. This needs to be explained, clearly and plainly, especially since this book claims to be aimed at students from multiple disciplines, not just computer science. (Both a recent number theory text [2] and a recent abstract algebra text [6] at hand use $\mathbb{N}$ for the positive natural numbers.)

I was not convinced by the informal introduction to proofs in Section 2.1. The basic story (as the authors tell it) is that before the 19th century, proofs were often deficient by modern standards. (Their example of a deficient proof is Fermat's proof of his last theorem. But since we don't have that proof, it's not an example of anything, let alone an example of a deficient proof.) The 19th century saw the development of modern, formal proof techniques. But these "ultimate-standard proofs made them quite unfriendly for humans to either craft or understand." My understanding is that this misstates the 19th century attempts at formalizing mathematics; that work was foundational work and was about assuring that proof was possible and that proofs were actually sound. I doubt that proofs of new theorems were ever required to use the Peano axioms. But nowadays, the authors argue, modern proofs are social exercises in which as long as everyone agrees it's a proof, then it's a proof. They give proof by induction as an example of a formalistic proof, and a proof using the pigeonhole principle as an example of a modern proof, saying "Are you convinced? If not, contact one (or both) of the authors, and we shall gladly provide more details." The pigeonhole principle example given seems to be a perfectly rigorous proof: all possible cases are enumerated.

I'm sympathetic to the author's desire "to overcome people's resistance to mathematical analysis and argumentation," but a "modern, human-friendly - but no less rigorous - methodology" strikes me as ultimately contradictory. There is a story to be told here, but telling it requires more care and thought.

Aside: there are at least two presentations of a proof of the infinitude of the primes (pp. 43-44 and pp. 237-238) that the authors seem to think is Euclid's proof but that differs from what other sources describe as Euclid's proof. Even worse, the authors incorrectly state "In fact, we claim n^* is a prime that is not in the sequence Prime Number." (Here n^* is 1 plus the supposedly finite product of all primes.) Uh, no. It's either prime or divisible by some number not in the claimed set.

Allow me to describe one of the problems I had with this book in detail. Chapter 3 has a section on "Boolean algebras." Having programmed in assembler on multiple architectures, I was

[4]https://www.youtube.com/c/MichaelPennMath

reasonably familiar with Boolean algebra. I thought. But even a rereading of the section left me confused. Consulting a variety of other texts (including Boole [1] himself) cleared up the problem. The authors had failed to explain that Boole developed (in the early 19th century) an algebra with radically different properties from ordinary algebra[5], and that that algebra turns out to be formally the same as the algebra of sets (which was developed in the late 19th century).

Since Chapter 3 is still introductory in nature, the authors chose not to introduce enough abstract algebra to describe this in its actual historical and logical structure, but rather chose to introduce set algebra first and then base the rest of the chapter on those ideas. Other textbooks describe the mathematics. In *Discrete Mathematics with Proof* [3] on p. 59 Gossett writes "A Boolean algebra, $\mathcal{B}$, consists of an associated set, B, together with three operators and four axioms." and then describes the components with examples. Gossett's example of using a power set as the carrier set makes the formal identity between Boole's algebra on truth values and Boolean algebras on sets blindingly clear. Getting this right takes less than two pages, including examples, if one has already presented the basics of abstract algebra.

Moving on from sets to propositional logic, the authors write "As we describe and define the basic connectives of the Propositional Logic, we point out their relationships to the Boolean set-related operations introduced in Section 3.2.2." So the bottom line is that the authors chose to present sets first, and operations on truth-valued variables as something related to those set operations, although that's logically and historically backwards. Allow me to quote the authors again: "Boole is generally credited with inventing these Boolean algebras." You, dear reader, may think that I'm overreacting here, but to me, this is simply wrong; Boole explicated the first of this class of algebras. He figured it out first, and gets his name on them because of that. The authors also write "... Boole is historically credited with developing the system we are describing here, with the goal of encapsulating a simple version of mathematical logic within an algebraic framework." I find these strangely insulting to Boole, who discovered the basic principles with which all modern digital computers operate.

In my first reading of Chapter 3, I thought I was still reading introductory material, and many of the things I see as problematic in Chapter 3 are a result of the authors planning to cover this material in more formal detail in later chapters. The presentation could function as a review of this material, but as either an introduction or as a textbook that expects the students to master the material, it strikes me as inadequate. Furthermore, Chapter 3 fails to explain its intellectual approach. Sets are, according to the chapter title, "The Stem Cells of Mathematics," yet the reader is left to infer for herself what that means. Ditto for "Structured Sets," which covers relations and functions (and some other things) described using set-theoretic notation. While I found it an interesting challenge to try to figure out what the authors had in mind, an undergraduate first encountering this material may be less amused.

Aside: The use of predicate logic in proofs is hard to explain. In everyday English, "implies" does a lot of work; in particular, it feels as though causality is being stipulated. But in predicate logic, it's just another binary operator on truth-valued variables. The authors, to their credit, discuss this. Several other introductions to the use of logic in proofs I've read failed to address this issue. So, kudos. This shows that the authors' idea that there is a need for the book this one tries to be is quite correct.

[5] Pinter [5] calls it "An even more bizarre kind of algebra".

3 What I Wrote in the Margins

The authors present a proof of Fermat's Little Theorem. This is a powerful tool that can obliterate seemingly impossible problems (such as determining the last 2 digits of 3^{400} [2], p. 55) in a flash. But the authors give no examples of its use. Thus the marginal notation "Examples, please!". On the same page I also wrote "Number of strings of length p over an alphabet of size $a = a^p$ and how that's equal to a mod p", which I thought was needed to understand the proof[6]. Which on the next page defines the "period" to be one less than the number needed to replicate the prior state (a shifted word in this case). The proof seems fine. It's just that using "period" to mean "one less than the what anyone else would see as the period" irritated. In a larger expression, the authors wrote $(i, i+1 \bmod n)$ and I wrote "an edge from node i to node $i+1$". My point here is that a lot of the work of reading this book lies in figuring out what the authors intended, and that they perhaps should have done more of that work. Maybe. The reader is allowed to think that some of this is my fault, not that of the book. But, I submit, if I need more help, undergraduate students will as well.

Another example: I scribbled "Doesn't define spanning tree." The section describes a spanning tree as being a graph with the same nodes and a subset of the edges, but fails to discuss what spanning means or why removing edges necessarily can create a tree. Other texts, e.g., [3], do. The things it does say about spanning trees are useful, interesting, and true. But they left out the definitions. Again, is this a quibble? I think not, because, overall, this book too often fails to provide definitions, either ahead of where they are needed by the reader, or at all. Thus the book comes across as inadequately user-friendly.

4 Quality of the Writing

The writing is enthused. The authors really like this material. Thus there are exclamation marks, superlatives, and metaphors throughout the text. In my opinion, this is excessive. My concern here is that while this writing is perfectly comprehensible to me, I worry that it could be found seriously irritating by the main target audience for this book: post-Millennial college undergraduates in the 2020s. Furthermore, many of them will be speakers of English as a second language. Words such as "betoken" will make an already linguistically difficult field that much more difficult. I'm not a fan of the term "user-friendly" but the lack thereof in this text is problematic.

There are two specific problems with the writing. The lesser of these problems is that the writing is outdated. The authors are fond of capitalizing things that shouldn't be capitalized. They write "the Theorem tells us that" when "this theorem tells us that" should be used. Terms such as propositional logic are capitalized, which, to the best I can tell, is no longer standard usage. There is also an overuse of double quotes. Almost every time something appears in double quotes, it means that the sentence in question needs to be rewritten. Randomly opening the text I found 'we now provide a "peek" into that area by means of...'. Inversely, the authors' use of italics is fine: it effectively brings out the point intended. The authors also seem overly fascinated by Latin, in at least one case using a Latin plural form. Again, this is not acceptable when your audience may include ESL speakers.

The more serious of the problems is that the writing is overly enthused, overly metaphorical,

[6] Yes, I realize that this sort of marginal note reflects my efforts at figuring out the proof, not necessarily a problem with the text.

and overly trite. Regarding infinitesimals, the authors write "The question of earliest discovery is one of the great real-life mysteries of all time." Uh, no. It isn't. On p. 91 the authors write: "In the lingo of the cognoscenti (the 'in-crowd'), these expressions are said to be in POS form, shorthand for (logical) product of (logical) sums." This particular example is, of course, embarrassingly bad in the extreme. Someone should have told the authors that.

5 Conclusions

There are a lot of things I like about this book, so I'm not happy that this sounds like such a negative review. It covers an amazing amount of mathematics in great detail. And that detail means that, if you work through the examples, you will have the preparation you will need for an upper level class, for example, one based on [4]. In addition, the later chapters, on combinatorics and graphs, provide an excellent introduction and preparation for further study. And even more challenging material is presented in the appendices. The bottom line, though, is that it's a flawed gem that needs work. In particular it needs two things. First, it needs an editor to bring the language up to ordinary, modern textbook standards and to reduce the rhetorical excesses. Second, it needs to be more generous with more detailed explanations of the technical terms, proofs, and derivations. As it is, I think that this book would be difficult for college sophomores. These, however, are low-level criticisms. The basic concept of the book, the overall structure of the book, and the material presented are all excellent: it deserves a better implementation.

References

[1] Boole, G., *The Complete Works of George Boole*, Shrine of Knowledge, 2020.

[2] Dujella, A., *Number Theory*, Školska knjiga, 2021.

[3] Gossett, E., *Discrete Mathematics with Proof*, Wiley, 2nd. ed., 2009.

[4] Graham, R. L., Knuth, D. E., Patashnik, O. *Concrete Mathematics*, Addison-Wesley, 2nd, ed., 1994.

[5] Pinter, C. C., *A Book of Abstract Algebra*, Dover, 2010.

[6] Weintraub, S. H., *Introduction to Abstract Algebra: Sets, Groups, Rings, and Fields*, World Scientific Publishing, 2022.

Comments on
Review by David J. Littleboy of our book
Understand Mathematics, Understand Computing

Arnold L. Rosenberg and Denis Trystram

We thank Mr. Littleboy for the substantial work of reviewing out 550-page book. We wish to comment on several aspects of his review. Our comments focus on broad issues relating to our book which we hope will provide the potential reader with helpful perspective.

Based on our long experience in teaching Discrete Maths both at the undergraduate and graduate level, we decided to write a textbook that does not correspond to writing in the historical sense. We do not aspire to teach Mathematics as a compendium of facts and tools but, rather, as a way of thinking and communicating and reasoning. We view readers as apprentices (initially) and collaborators (eventually). This approach gives our book the feel of a coherent tapestry of ideas which interconnect in often quite-nonlinear ways. The "forward" references that Mr. Littleboy mentions with discomfort intentionally suggest to the reader that what we call "doing" mathematics is a process wherein there is often more to say about a subject as we develop more background and intuition. We always strive to keep the reader apprised of the path we are following, via appropriate references and extensive discussion. In addition to being teachers, we play the role of guides as we develop the various mathematical topics our book contains. We help assimilate the readers (especially the most junior ones) by always providing readers several proofs of important results—indeed, several *types of proofs* built on a combination of text (for the textual thinker) and figures (for the non-textual thinker). We go far beyond the practice of other textbooks in these regards, as we help readers to enter the abstract world of mathematics.

1 Mathematics as a Way of Thinking

Mathematics is far more than a collection of facts generated by a set of concepts enhanced by tools for manipulating the concepts. Deep philosophical issues abound and connect us to the ontological foundations (i.e., true nature) of *mathematical "objects"* such as numbers, functions, relations (naming just a few)—and of the representational aspect of these objects—which *importantly* are what we compute with. A few examples will suffice:

- Exemplifying *mathematical "objects"*:
  - What is "nothing"? How does the concept *zero* represent "nothing"? Are there multiple candidates for a *zero*, which capture this concept in distinct ways?
  - At the other end of the spectrum, what is "infinity"? In what ways does inifinitude differ from finitude? Is there more than one valid — i.e., logically consistent — notion "infinity"?
- Exemplifying the *mechanisms that underlie mathematical reasoning*: Formalizing hypotheses, decomposing arguments into steps, invoking logical inference, and writing the proof that is our ultimate goal.

  - What is the essence of logical reasoning? of logical argumentation?
  - What does it mean to say that one proposition *implies* another?
  - When has one established that two propositions are "equivalent", in the sense that logical arguments cannot distinguish them?

The preceding are, of course, *foundational* questions, but each has *operational* analogues, as suggested by the following observations.

A crucial step in doing mathematics can be termed *modeling*: developing mechanisms for explicit reasoning, formalizing hypotheses, decomposing complex phenomena into simpler components, and making the logical inferences that ultimately lead one to mathematical proofs about real phenomena.

- Exemplifying the described processes and goals:

  Over the millennia, people have developed mathematical systems – collections of objects (sets, numbers, etc.), with relations that expose connections among objects, and with repertoires of operations that transform the objects. Much can be learned from studying such systems, including remarkable equivalences such as:

  - The following mathematical systems are "essentially equivalent", in a sense that can be formalized mathematically:
    * The system of *set algebras* – sets with operations such as union and intersection.
    * The system of *logical calculi* – logical formulae with operations such as conjunction and disjunction.
    * The system of *digital logical design* – circuit elements with operations such as **and** and **or**.

    Moreover, all of these systems form *Boolean algebras*.
  - The following computational problems are "essentially equivalent computationally," in the sense that an efficient solution-algorithm for any of these problems can be efficiently transformed into an efficient solution-algorithm for any of the others.
    * The problem of coloring the vertices of a given graph, using the fewest colors possible, in such a way that neighboring vertices have distinct colors.
    * The problem of scheduling meetings in the fewest individual rooms that guarantees privacy.
    * The problem of deciding whether a given graph can be drawn long a line in such a way that each pair of adjacent vertices are connected by an edge in the graph.

2 Mathematics within History and Culture

Systems such as Mathematics do not arise within a vacuum. We take pleasure in the book in discussing with the readers interesting historical/cultural aspects of Mathematics, even as we expose them to the technical aspects of the field. The aspects we allude to include linguistic and cultural origins of various mathematical concepts and terms. Examples include the Latinate and Greek origins of many terms and concepts, as well as the origin of the word "algorithm" and of historically named systems such as our Hindu-Arabic number system.

3 Explanation via Stylistic Convention

We have decided to honor the names of fields of inquiry via capitalization. This practice sometimes makes distinctions such as "Computer Engineering as a field" vs. "computer engineering as an activity" recognizable without distracting comments.

Similarly honoring the names of theorems, lemmas, etc., under discussion relieves the reader of the chore of decoding authors' expositional intentions from their use of articles, such as "the" vs. "this".