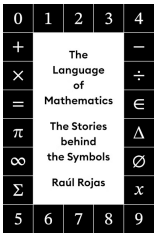
Review of [1]

The Language of Mathematics

The Stories behind the Symbols

Raúl Rojas (translated by **Eduardo Aparicio**)



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Review by
Nicholas Tran (ntran@scu.edu)

This book traces the origins of more than 60 familiar symbols in the mathematical alphabet, providing interesting details about the lives of the people who introduced and popularized them in the process. Written in Spanish by the prize-winning computer scientist and historian Raúl Rojas and translated into English by Eduardo Aparicio, the book is an easy and enjoyable read for a general audience, requiring no technical background. Mathematicians will appreciate reproductions of original manuscripts where the symbols first appeared, whereas nonspecialists may find inspiring the author's observations about these symbols and their inventors (e.g., "[the universal quantifier ∀] resembles a cubist tear flowing from an eye that Picasso could have painted.") There are nine chapters with fifty-four self-contained sections, loosely organized by topic and in chronological order.

1 Summary

Chapter 1 traces the birth of algebra and the need for symbols back to Diophantus and al-Khwārizmī. It presents frequency counts of the top twenty letters and mathematical symbols that appear in *arXiv* mathematical texts and some engineering textbooks as evidence of their importance in modern mathematical writing. To further demonstrate the power of symbols to express deep ideas concisely, it presents the three most beautiful formulae in mathematics according to a recent survey: Pythagorean theorem ($a^2 + b^2 = c^2$), Euler's identity ($e^{i\pi} + 1 = 0$), and Euler's formula for polyhedra ($V - E + F = 2$). The chapter ends with a discussion of why we should refer to *sides* of an equation instead of its *roots*.

Chapter 2 is about number systems and variables. The evolution of our modern positional decimal system, from the Babylonian base-60 system to the Hindu base-10 system with zero added, and later to the Arabic system with the decimal point, is described, along with the base-20 Mayan system used throughout Mesoamerica. The chapter also discusses the use of letters as variables, starting with Fibonacci pioneering the use of letters to denote numbers. François Viète adopted the use of Latin consonants for constants and Latin vowels for variables. His countryman René Descartes later changed to using letters at the beginning of the Latin alphabet for constants and

letters at the end for variables, a convention that has prevailed to this day. The chapter also discusses adorning variables with superscripts and subscripts, the absolute value bar, and the life of its inventor Karl Weierstrass.

Chapter 3 points out the use of two different types of cross symbols for arithmetic operators (and three in the British flag). It also describes the formal theory of negative integers as pairs of natural numbers and explains the German saying "nach Adam Riese" ("according to Adam Ries") referring to the father of modern calculating.

Chapter 4 presents short biographies of Robert Recorde, who invented the equal sign (=), and Thomas Harriot, who introduced the greater-than and less-than signs ($>$ and $<$). The chapter also recounts the triumph of the parentheses over the vinculum ($\overline{}$) for grouping symbols. It ends with a discussion of the history of the comma and the period as separators.

Chapter 5 delves into symbols in calculus, including the integral sign $\int$ and the differential symbol d, both introduced by Gottfried Leibniz; the partial derivative symbol ∂ created by Adrien-Marie Legendre; the nabla symbol ∇ invented by William Hamilton; the infinity symbol ∞ proposed by John Wallis; the function notation $f(x)$ popularized by Leonhard Euler; and the limit notation $\lim_{x \to a}$ introduced by John Leathem. The chapter also discusses the formal models of real numbers developed by Richard Dedekind and Georg Cantor.

Chapter 6 focuses on symbols in set theory and logic, such as the element-of symbol $\in$ and existential quantifier $\exists$ introduced by Giuseppe Peano; the universal quantifier $\forall$ invented by Gerhard Gentzen; the empty set symbol $\emptyset$, the integer set symbol $\mathbb{Z}$ and the rational set symbol $\mathbb{Q}$ created by the Bourbaki group. The chapter gives a brief explanation of Ernst Zermelo-Abraham Fraenkel's axiomatization of set theory and Georg Cantor's diagonal argument that gave rise to the hierarchy of infinities $\aleph_0 < \aleph_1 < \aleph_2 < \cdots$.

Chapter 7 examines symbols for constants such as e (Euler's number or Napier's constant), π (Archimedes's constant or Ludolphian number), i ($\sqrt{-1}$), h (Planck's constant), c (speed of light), G (gravitational constant), ε_0 (Coulomb constant) and k_b (Boltzmann constant).

Chapter 8 surveys symbols in discrete mathematics and combinatorics, including the factorial symbol ! introduced by Christian Kramp, the summation symbol $\sum$ popularized by Leonhard Euler, the binomial coefficient symbol $\binom{n}{k}$ proposed by Andreas von Ettingshausen, and the floor and ceiling symbols $\lfloor x \rfloor$ and $\lceil x \rceil$ created by Kenneth Iverson.

Chapter 9 concludes with a discussion of miscellaneous symbols such as the QED symbol ■ invented by Paul Halmos, the congruence symbol $\equiv$ introduced by Carl Friedrich Gauss, and the matrix notation using square brackets popularized by Arthur Cayley. It also explains the surprising origins of the terms sin, cos, and tan.

2 Opinion

The book manages to be both scholarly and entertaining at the same time, a rare feat for mathematical writing. As noted in the foreword, certain details appear several times to ensure each section is self-contained, but this repetition does not diminish the overall reading experience. I think the book would be excellent supplementary reading for an introductory course in calculus or discrete mathematics, as it provides historical context, humanizes the subject matter, and offers lucid, informal explanations of some mathematical concepts often found difficult by undergraduates such as limit, Dedekind cut, δ-ϵ definitions, diagonal argument, and axiomatic set theory.