The Book Review Column [1]

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1 Notable new releases

Foundation Mathematics for Computer Science: A Visual Approach, 4th edition (Springer, 2023) by John Vince (Bournemouth University, UK) is a comprehensive collection of discrete and continuous mathematical topics that are covered in most undergraduate programs in computer science. The subtitle refers to the author's use of colored graphs and tables to illustrate the concepts.

Online Algorithms (Cambridge University Press, 2023) by Rahul Vaze (Tata Institute of Fundamental Research, India) is an accessible but rigorous introduction to the area aimed at advanced undergraduates and beginning graduate students. The book covers the basic as well as applied online problems with a preference of elegant analysis over performance.

Privacy-preserving Computing for Big Data Analytics and AI (Cambridge University Press, 2023) by Kai Chen and Qiang Yang (Hong Kong University of Science and Technology) is a systematic examination of the history, theories, techniques, applications, and future of the field.

Prize-winning neuroscientist Terrence Sejnowski (University of California at San Diego) explains the technology and mathematics behind large language models such as ChatGPT and explores the debate on their so-called comprehension of language in *ChatGPT and the Future of AI: The Deep Language Revolution* (The MIT Press, 2024).

2 This column

A mathematics book that starts with a poem by Jacques Prévert and five forewords deserves closer examination. Bill Gasarch reviews *The New Mathematical Coloring Book* by Alexander Soifer, a genre-breaking work that covers graph coloring, chromatic number of the plane, Ramsey theory, history, logic, and more. The book is an updated and expanded sequel to Soifer's earlier book, *The Mathematical Coloring Book*, which Bill reviewed in this column in 2009.

Looking for an accessible introduction to modern developments in coding theory? Rutuja Kshirsagar and Gretchen Matthews highly recommends *Essays on Coding Theory* by Ian Blake for its broad coverage of the field and its engaging and lucid writing style.

Students of *Winning Ways for your Mathematical Plays* will be interested in S. V. Nagaraj's précis of *Combinatorial Game Theory: A Special Collection in Honor of Elwyn Berlekamp, John H. Conway and Richard K. Guy* edited by Richard Nowakowski, Bill Landman, Florian Luca, Melvyn Nathanson, Jaroslav Nešetřil, and Alex Robertson. The book is a tribute by game theorists to the three masters in the field.

3 How to contribute

This holiday season, consider gifting the SIGACT community with your review of a book. Either choose from the books listed below, or propose your own. In either case, the publisher will send you a free copy of the book. Guidelines and a LaTeX template can be found at `https://algoplexity.com/~ntran`.

BOOKS THAT NEED REVIEWERS FOR THE SIGACT NEWS COLUMN

Algorithms, Complexity, & Computability

1. Vaze, R. (2023). *Online Algorithms.* Cambridge University Press.
2. Ferragina. P. (2023). *Pearls of Algorithm Engineering.* Cambridge University Press.
3. Downey, R. (2024). *Computability and Complexity: Foundations and Tools for Pursuing Scientific Applications.* Springer.

Miscellaneous Computer Science & Mathematics

1. Grechuk, B. (2019) *Theorems of the 21st Century.* Springer.
2. Chayka, K. (2024). *Filterworld: How Algorithms Flattened Culture.* Doubleday.
3. Valiant, L. (2024). *The Importance of Being Educable: A New Theory of Human Uniqueness.* Princeton University Press.

Data Science

1. Sejnowski, T. (2024). *ChatGPT and the Future of AI: The Deep Language Revolution.* The MIT Press.

Discrete Mathematics and Computing

1. Ross, S., & Peköz, E. (2023). *A Second Course in Probability.* Cambridge University Press.
2. Vince, J. (2024). *Foundation Mathematics for Computer Science: A Visual Approach, 4th edition.* Springer.

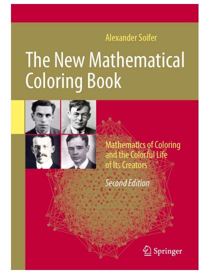
Cryptography and Security

1. Chen, K., & Yang, Q. (2023). *Privacy-preserving Computing for Big Data Analytics and AI.* Cambridge University Press.
2. Garfinkel, S. (2025). *Differential Privacy.* The MIT Press.

Combinatorics and Graph Theory

1. Landman, B., Luca, F., Nathanson, M., Nešetřil, J., & Robertson, A. (Eds.). (2022). *Number Theory and Combinatorics: A Collection in Honor of the Mathematics of Ronald Graham*. De Gruyter.

Review of [1,2]



The New Mathematical Coloring Book:
Mathematics of Coloring and the Colorful Life of its Creators
Second edition
by **Alexander Soifer**

Springer, 2024
eBook $169, Hardcover $220, 889 pages

Review by **William Gasarch** (`gasarch@umd.edu`)

1 The NEW Mathematical Coloring Book

In 2009 Alexander Soifer published *The Mathematical Coloring Book*. I will refer to this book by TMCB. TMCB is around 600 pages. I reviewed it for SIGACT News here:

`https://www.cs.umd.edu/~gasarch/bookrev/40-3.pdf`.

In 2024 Alexander Soifer published *The New Mathematical Coloring Book*. I will refer to this book by TNMCB. TNMCB is around 900 oversized pages.

In this review I first republish the first two sections of my review of TMCB (with a few comments added), then summarize what's common to both books, and what's new in TNMCB. I will then render an opinion.

Begin Excerpt from Prior Review

2 Introduction

I first had the pleasure of meeting Alexander Soifer at one of the *Southeastern International Conferences on Combinatorics, Computing, and Graph Theory*. If I was as careful a historian as he is, I would know which one. Over lunch he told me about van der Waerden's behavior when he was living as a Dutch citizen in Nazi Germany. Van der Waerden later claimed that he opposed the firing of Jewish professors. Soifer explained to me that in 1933 the German government passed a law requiring universities to fire all Jewish professors *unless they were veterans of WW I* (there were other exceptions also). Van der Waerden protested that veterans were being fired, in violation of the law. So he was objecting to *the law not being carried out properly* and not to *the law itself*. Alex told me that the full story would soon appear in a book he was writing on coloring theorems. I couldn't tell if the book would be a math book or a history book. It is both.

BEGIN Added Comment

[2] This review first appeared in *Geombinatorics* 34(1), July 2024. It appears here with their permission.

Both TMCB and TNMCB have a lot about van der Waerden and his actions under the Nazi regime. Alexander Soifer has a separate book on that, *The Scholar and the State* [2]. I reviewed that book for SIGACT News here:

`https://www.cs.umd.edu/~gasarch/bookrev/FRED/vdwhistory.pdf`

END Added Comment

The second time I met Alex was at the next *Southeastern International Conference on Combinatorics, Computing, and Graph Theory.* Alex gave a talk on the following:

1. Prove that for any 2-coloring of the plane there are two points an inch apart that are the same color. (This is easy: I was able to do it in 2 minutes.)

2. Prove that for any 3-coloring of the the plane there are two points an inch apart that are the same color. (This is easy: I was able to do it in 3 minutes.)

   BEGIN Added Comment

   D.N.J. de Grey showed, in 2018, that for any 4-coloring of the plane there are two points an inch apart that are the same color. This proof requires the use of a computer. (This is hard: I was unable to do it in n minutes for all $n \in \mathsf{N}$.)

   END Added Comment

3. Prove that there is a 7-coloring of the plane such that for all points p, q that are an inch apart, p and q are different colors. (This is easy: I was able to do it in 7 minutes.)

4. Find the number χ such that (1) for any $(\chi - 1)$-coloring of the plane there will be two points an inch apart that are the same color, and (2) there exists a χ-coloring of the plane such that for all points p, q that are an inch apart, p and q are different colors. (This is open: I was unable to do this in χ minutes.)

Alex uses the symbol χ for this quantity throughout the book; hence I will use the symbol χ for this quantity throughout the review.

The problem of determining χ is called the *Chromatic Number of the Plane Problem* and is abbreviated CNP. Alex told me *CNP is his favorite problem in all of mathematics.* He especially likes that the problem can be explained to a layperson, yet leads to advanced mathematical concepts.

After seeing Alex's talk I asked my colleague Clyde Kruskal what happens if only a subset of the plane is colored. For example, what is the largest square that can be 2-colored? 3-colored? Clyde then obtained full characterizations of 2 and 3-colorings for rectangles and regular polygons [1]. The paper contains the following marvelous result: an $s \times s$ square is 3-colorable iff $s \leq 8/\sqrt{65}$.

After talking to Alex I very much looked forward to his book. I first got my hands on it at the SODA (Symposium on Discrete Algorithms) conference of 2009. The Springer-Verlag book vendor let me read parts of it during the coffee break. I later got a copy and read the whole thing.

3 What Kind of Book is this?

When I first read the book I noticed something odd. The first sentence is *I recall April of 1970.* **Most of the book is written in the first person, like a memoir or autobiography!** The only parts that are not written in first person are when someone else is doing the talking.

In Alex's honor my review is written in his style.

Ordinary math books are not written in the first person; however, this is no ordinary math book! I pity the Library of Congress person who has to classify it. This book contains much math of interest and pointers to more math of interest. All of it has to do with coloring: coloring the plane (Alex's favorite problem), coloring a graph (e.g., the four color theorem), and of course Ramsey Theory. However, the book also has biographies of the people involved and scholarly discussions of who-conjectured-what-when and who-proved-what-when. When I took calculus the textbook had a 120-*word* passage about the life of Newton. This book has a 120-*page* passage about the life of van der Waerden.

Is this a math book? YES. Is this a book on history of Math? YES. Is this a personal memoir? YES in that the book explicitly tells us of his interactions with other mathematicians, and implicitly tells us of his love for these type of problems.

Usually I save my opinion of the book for the end. For this book, I can't wait:

This is a Fantastic Book! Go buy it Now!

BEGIN Added Comment

This comment was made of TMCB but it also holds for TNMCB.

END Added Comment

End of Excerpt of Prior Review

4 Something Old, Something New, Something Borrowed, Something k-Colored

TMCB has 11 parts and 49 chapters. TNMCB has 13 parts and 68 chapters. For a detailed description of what is in TMCB, see my review. For now, I list topics that are in both books and how many chapters each book devotes to them.

Topic	TMCB	TNMCB
Chromatic Number of the Plane	15	30
Vertex and Edge Colorings of a Graph	11	13
Ramsey Theory	10	11
History	8	9
Logic	3	3
Miscellaneous	2	2

The above chart was about *chapters*. I now discuss *parts*. There are two new parts:

1. **Ask what your computer can do for you.** This is approximately 50 pages of completely new material. It covers the proof that $\chi \geq 5$ by de Grey and much of the work that it inspired.

2. **What About Chromatic 6?** This is approximately 30 pages of completely new material. Now that $\chi \geq 5$ is known, what about $\chi \geq 6$? This chapter does not answer that question; however, it gives many related results.

The list above does not capture the breadth and depth of TNMCB because (1) some chapters are hard to classify, and (2) some chapters are in both books, but there is a lot more in TNMCB.

I describe some of the topics that are in TNMCB but not in TMCB.

4.1 The Chromatic Number of the Plane

Let χ be the least number so that there is a χ-coloring of the real plane such that no two points of the same color are an inch apart. For 68 years it was known that $4 \le \chi \le 7$. While people (including Soifer) studied variants of the problem, there was no progress on the original problem.

Until 2018.

In that year D.N.J. de Grey showed that $\chi \ge 5$. The proof used a computer program. Soifer gives the history and context of the result, since he had the best seat in the house to the events.

Soifer shares his opinion of what χ is. Before de Grey's result Soifer thought $\chi = 7$. He still thinks so. In fact, he has a more general conjecture. Let χ_n be the chromatic number of E^n where we connect two points iff they are an inch apart. Soifer thinks $\chi_n = 2^{n+1} - 1$. Note that this conjecture implies $\chi_2 = 7$.

4.2 Finite Sets Have A Role. Or Do They?

De Bruijn and Erdös (1951) proved that, for any graph G, G is k-colorable iff every finite subset of G is k-colorable. (Today this would be called a standard compactness argument; however, like many things it was harder then but easy now.) Hence there is a finite number of points in the plane such that χ is the chromatic number of the unit-distance graph they form. This proof uses the Axiom of Choice. This result takes the book in two directions.

Unit-Distance Graphs of Girth $\ge X$

Consider the following finite sets of points.

1. Three points of an equilateral triangle of side 1. This finite set shows that $\chi \ge 3$.

2. The Mosers Spindler is a 7-point set that shows $\chi \ge 4$. (Note that its *Mosers* not *Moser*. That is because two brothers, Leo and William Moser, came up with it together.) These seven points have 4 triangles of side 1.

A *unit-distance graph* is a set of points in the plane where if two of them are 1-apart, we put an edge between them.

Noting that the Mosers Spindle is a 4-chromatic unit-distance graph with triangles, Paul Erdös, in 1975, posed the following problem:

Is there a 4-chromatic unit-distance graph of girth 4,5 or higher?

In 1979 Nicholas Wormald constructed a 4-chromatic unit-distance graph of girth 5 on 6448 vertices. In 1990 Alexander Soifer, in his journal *Geombinatorics*, asked for the smallest such graph. This led to a flurry of results by different people culminating (for a time) in the following which are described in TMCB:

1. In 1996 Paul O'Donnell and Rob Hochberg obtained a 23-node 4-chromatic unit-distance graph with girth 4. This was a joint paper.

2. In 1996 Paul O'Donnell and Rob Hochberg obtained a 45-node 4-chromatic unit-distance graph with girth 5. This was a joint paper.

TNMCB describes the following new results:

1. In 2016 Exoo-Ismailescu obtained a 17-node 4-chromatic graph with girth 4, and proved that 17 is the best possible.
2. Because of de Grey's result it now made sense to look for 5-chromatic unit-distance graphs of small order. This challenge led to a flurry of results, including results by the following two people:
   (a) In 2018 Marijn Heule had a series of results culminating in a 5-chromatic unit-distance graph on 510 vertices.
   (b) In 2020 Jaan Parts obtained a 5-chromatic unit-distance graph on 509 vertices.

The two graphs have girth 3. Soifer asks in the book for the smallest 5-chromatic unit-distance graph of girth 4.

Full details are given plus lots of color pictures of graphs. And much like de Grey's result, Soifer offers an insider view of both the math and the history.

Different Models of Set Theory

The Continuum Hypothesis is independent of ZFC (Zermelo-Frankel Set Theory with the Axiom of Choice), a system where one can do virtually all of mathematics except from some questions in logic—though we will come back to that point. Is it possible that the value of χ is independent of ZFC?

There are a few natural (we'll come back to that point too) problems that are independent of set theory. The most notable one is CH (the Continuum Hypothesis: is there a cardinality between countable and the reals). The independence of CH was shown in two parts: (1) Gödel showed there is a model where CH is true, and (2) Cohen showed that there is a model where CH is false. AC held in both models.

In looking at models where the value of χ might change, it may be useful to drop AC and replace it with something else. Is dropping AC a good idea?

Consider the graph $G = (V, E)$ where

- $V = \mathsf{R}$ (the reals),
- $E = \{(s,t) \in \mathsf{R}^2 : s - t - \sqrt{2} \in \mathsf{Q}\}$.

What is $\chi(G)$? The answer is not so simple.

ZFS is the set of axioms of $\mathrm{ZF} + \mathrm{AC}_{\aleph_0} + \mathrm{LM}$ where $\mathrm{AC}_{\aleph_0}$ is AC for countable sets and LM is the statement that all sets are Lebesgue measurable, hence guaranteeing no Banach-Tarski Paradox. The S stands for Solovay who proved that if ZFC is consistent then ZFS is consistent. Let $\chi^{\mathrm{ZFC}}(G)$ be $\chi(G)$ in ZFC, and let $\chi^{\mathrm{ZFS}}(G)$ be $\chi(G)$ in ZFS. Soifer and Shelah showed that

1. $\chi^{\mathrm{ZFC}}(G) = 2$
2. $\chi^{\mathrm{ZFS}}(G) > \aleph_0$.

These results were in TMCB. Between TMCB and TNMCB Soifer spoke to many logicians (Paul Cohen, Robert Solovay, and Saharon Shelah) and other mathematicians, about these results and what they mean.

Soifer then has a chapter on what he thinks. Most mathematicians are Platonists. They think that questions such as CH or the chromatic number of the G *have answers*. In short, mathematicians *imagine the real*. This is also what scientists do. To quote the book *Science reflects what is outside of the Man, in Nature, whereas Art reflects what is within.*

Soifer thinks of math as being an art. Hence, to quote the book, *Mathematics is an invention that makes us realize reality.* Soifer coined a term for this philosophy: *Imaginism.* He gives strong evidence that Einstein, Picasso, Wittgenstein, Baudelaire, and Camus were Imaginists.

4.3 How Can a Math Book be Controversial?

TMCB has a few chapters on history that examine van der Waerden's behavior during WW II. He stayed in Nazi Germany when he could have left. The treatment of him given here was nuanced and fair. When I read TMCB, I thought this material was interesting but not controversial.

I should have been right, but I was wrong.

Gunter M. Ziegler wrote a negative book review of TMCB. Soifer carefully refutes everything that Ziegler wrote. But why was Ziegler so negative? I thought that Soifer was going to say that even bringing up Germany's Nazi past was considered controversial. While that is surely true, there is more going on here.

The criticism of TMCB came after Soifer went on a campaign to rename *The Nevanlinna Prize*. Why rename it? Because Nevanlinna was a Nazi collaborator. Soifer speculates that the campaign brings up Germany's Nazi past, and that is what Ziegler objects to.

TNMCB also gives a careful account of the campaign. Spoiler alert: the name was changed to *The Abacus Medal*. That's of course good, but there were other issues involved. Soifer discusses all of this.

Nevanlinna was a fine analyst but had no theoretical computer science credentials. I wonder if the name change would have happened if Nevanlinna was a brilliant theoretical computer scientist.

4.4 Pictures of People, Graphs, and Letters

TNMCB has the following

1. Many color pictures of mathematicians and others.
2. Many color pictures of graphs.
3. Some copies of handwritten letters that are of interest for history.

4.5 Other

I have given an account of changes that lead to new chapters or half-chapters. There are many new sections as well that are harder to summarize.

5 Opinion

(This is the same Opinion I gave in my review of TMCB.)

Who *could* read this book? The upward closure of the union of the following people: (1) an excellent high school student, (2) a very good college math major, (3) a good grad student in math or math-related field, (4) a fair PhD in combinatorics, or (5) a bad math professor.

Who *should* read this book? Anyone who is interested in math or history of math. This book has plenty of both. If you are interested in math, then this book will make you interested in history of math. If you are interested in history of math, then this book will make you interested in math. Any researcher in either mathematics or the history of mathematics, no matter how sophisticated, will find many interesting things they did not know.

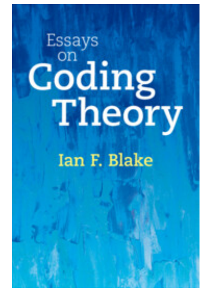
And now the elephant in the room: If you have TMCB, should you buy TNMCB? Yes. There is so much more here that is worth knowing.

References

[1] C. Kruskal. The chromatic number of the plane: the bounded case. *Journal of Computer and System Sciences*, 74:598–627, 2008. `www.cs.umd.edu/~kruskal/papers/papers.html`.

[2] A. Soifer. *The Scholar and the State: In Search of Van der Waerden.* Birkhäuser Springer-Verlag, Basel, 2015.

Review of [1,2]



Essays on Coding Theory
by **Ian F. Blake**

Cambridge University Press, 2024
Hardback $69.99, eBook $69.99, 474 pages

Review by **Rutuja Kshirsagar** and **Gretchen L. Matthews**

1 Overview

Coding theory, a branch of mathematics and theoretical computer science, focuses on the efficient sharing and storage of information. Since its inception in 1948, the field has made significant strides and now finds applications across various domains, including communications, data storage, cryptography, and security. The core challenges in coding theory can be broadly categorized into three main areas: 1. How to efficiently encode information or data; 2. How to transmit or share this encoded data; and 3. How to decode the received data to accurately retrieve the original information.

Ian F. Blake's book *Essays on Coding Theory* provides an in-depth review of various topics within the field. Aimed at graduate students and early-career researchers, the book is designed to help readers grasp the fundamentals of modern coding theory - topics that do not appear in more standard texts. Although it assumes a foundational understanding of algebraic coding and information theory, important definitions and terminologies are introduced either in the introductory chapter or at the start of each chapter as needed. The author has made a commendable effort to maintain consistent notation throughout the book, enhancing readability. The explanations are clear, and while some proofs are not presented in full detail, the author effectively guides readers through the proof strategies. The primary goal of the book is to acquaint readers with research in different areas of coding theory, rather than presenting the most current advancements. Nevertheless, the book includes references to more recent publications and resources for those interested in further exploration.

2 Book Summary

The book is organized in the following way:

[2] Another review of this text by the same authors appears in the American Mathematical Monthly, published online: 7 Oct 2024.

- **Chapter 1 and 2** address fundamental concepts such as finite fields, error correction, and erasure recovery. These foundational topics are crucial for building a robust understanding of the more advanced ideas presented throughout the book. By thoroughly explaining these core principles, the author ensures that readers are well-prepared to grasp the subsequent, more complex theories and applications discussed in the later chapters.

- **Chapter 3** focuses extensively on low-density parity-check (LDPC) codes, a class of error-correcting codes first introduced by Gallager. LDPC codes are renowned for their remarkable error-correcting performance and efficiency, which make them invaluable across a range of applications, including the advancement of flash memory technologies. The chapter not only explores the foundational concepts established by Gallager but also covers subsequent developments and refinements in LDPC codes. These advancements include improvements in decoding algorithms, optimization techniques, and novel applications that have further enhanced the practicality and performance of LDPC codes in modern communication systems.

- **Chapter 4** explores polar codes, a class of error-correcting codes introduced by Arikan in 2008. These codes are particularly significant for their role in the development of 5G networks. Polar codes are distinguished by their advanced error-correcting capabilities, which play a crucial role in ensuring the efficient and reliable transmission of data in modern communication systems. The chapter provides the theoretical foundations of polar codes and their construction principles.

- **Chapter 5-10** details the concepts of locality and distributed storage, highlighting the importance of efficiently recovering and managing data. The chapters examine various families of codes that leverage the principle of accessing a small subset of code symbols to correct errors or retrieve missing information. Key code families covered include locally recoverable codes, locally decodable codes, regenerating codes, network codes, and batch codes, among others.

  Each of these code families employs innovative techniques to enhance data reliability and accessibility, which are crucial for modern distributed storage systems. Locally recoverable codes, for instance, allow for the recovery of lost or corrupted data by accessing only a small number of other code symbols. Locally decodable codes enable the retrieval of specific data bits without needing to decode the entire dataset, while regenerating codes focus on optimizing the repair of lost data. Network codes and batch codes further contribute to efficient data management and error correction across distributed systems.

  Chapter 9 addresses private information retrieval (PIR) and PIR storage. The central concept remains the efficient access to localized information, but with an added dimension of privacy. In PIR, the retrieval of information is conducted in such a way that the server from which the data is downloaded remains unaware of the specific information being accessed. This ensures that the retrieval process is not only efficient but also preserves user privacy by concealing the nature of the data request from the server.

  Overall, these chapters provide a comprehensive overview of how these advanced coding techniques are applied to improve data recovery and storage in distributed systems, showcasing their significant role in advancing modern information technology.

- **Chapter 11** introduces graph-based codes, including Tanner codes developed by Tanner and expander codes created by Sipser and Spielman. Expander graphs, particularly Ramanujan graphs, are notable for their strong connectivity and sparsity. These properties enable the development of codes with efficient encoding and decoding algorithms.

- **Chapter 12** explores coding methods that extend beyond traditional codes defined over finite fields with distance measured by the Hamming metric. It examines rank-metric codes, initially introduced by Delsarte, where the distance between codewords is defined by the rank of a matrix rather than by Hamming weight. Additionally, the chapter covers subspace codes, where codewords are subspaces of a vector space, and the distance is defined by a relevant metric on these subspaces. The chapter primarily focuses on developing and analyzing these advanced coding concepts.

- **Chapter 13** describes list decoding algorithms developed by Sudan and Guruswami. In contrast to traditional decoding methods that produce a single decoded result, list decoding algorithms provide a list of possible solutions. The chapter covers a range of list decoding techniques and their practical applications, highlighting how they enhance error correction. Furthermore, it explores the influence of list decoding on the design of capacity-approaching codes, demonstrating how these techniques contribute to developing codes that approach the theoretical limits of channel capacity.

- **Chapter 14** focuses on methods for generating sequences with specific desirable properties. These specialized sequences are instrumental in the development of codes used in various applications, including communication systems, mobile phones, and space technologies. The chapter reviews techniques for constructing these sequences and examines their significance in enhancing the performance and reliability of coding schemes across different fields.

- **Chapter 15 and 16** represent a gradual shift from coding theory to the realm of cryptography and quantum computing. They introduce key concepts in cryptography, with a particular focus on various schemes from post-quantum cryptography, highlighting their relevance in the context of emerging quantum computing technologies.

  These chapters provide an overview of the fundamental principles of quantum computation, explaining why quantum computing poses challenges for traditional cryptographic methods. They also cover essential concepts in quantum error correction (QEC), outlining the fundamental principles and the role of QEC codes in correcting errors in quantum systems.

  While the chapters introduce some small QEC codes to illustrate these concepts, they do not delve deeply into a broad range of QEC codes. Instead, they offer a foundational understanding of quantum error correction, setting the stage for readers to appreciate the interplay between quantum computing and cryptographic security without overwhelming them with excessive technical details.

- **Chapter 17** explores additional types of codes, including balanced codes and permutation codes, among others. This diverse coverage ensures a comprehensive conclusion to the book, rounding out the discussion with a broad overview of various coding techniques. By including these additional code families, the chapter effectively wraps up the book, providing readers with a well-rounded understanding of the field.

- **Appendices** provide essential background on algebraic concepts that are crucial for understanding the material presented in the book. They cover finite geometries, linearized polynomials, Gaussian coefficients, Hasse derivatives, and the zeros of multivariate polynomials. These topics are fundamental for grasping the more advanced concepts discussed throughout the text.

3 Opinion and Conclusions

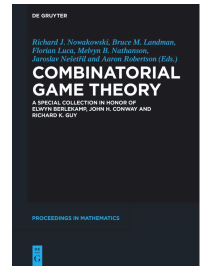
Blake's book is truly commendable and deserves high praise. We highly recommend it to anyone interested in modern coding theory, as it covers a broad range of current topics with reasonable depth and clarity. This book would be particularly valuable for professors teaching graduate or specialized courses on coding theory, providing a solid foundation and extensive resources for both teaching and further study. The book remains an excellent resource and a highly engaging read. Its comprehensive coverage and clear explanations make it a significant contribution to the field, and it will undoubtedly be a valuable asset for both students and educators alike. In summary, Blake's work stands out as a pivotal text that not only enhances understanding but also fosters a deeper appreciation for the complexities and innovations in coding theory.

Review of[1]

Combinatorial Game Theory:
A Special Collection in Honor of
Elwyn Berlekamp, John H. Conway and Richard K. Guy

by **R. J. Nowakowski**, **B. M. Landman**, **F. Luca**,
M. B. Nathanson, **J. Nešetřil**, and **A. Robertson** (Eds.)

De Gruyter, 2022
Hardcover $230, eBook $230, 430 pages



Review by
S.V.Nagaraj (`svnagaraj@acm.org`)
Vellore Institute of Technology, Chennai Campus, India

1 Introduction

This book is devoted to the remembrance of the work of the three mathematicians: Elwyn Berlekamp, John Conway, and Richard Guy. It includes twenty writings from researchers reflecting on their work in combinatorial game theory that should be of interest to researchers and students working in the area.

2 Summary

The book comprises twenty chapters. The first chapter is on the game of Flipping Coins, a partizan version of the game Turning Turtles introduced by Berlekamp, Conway, and Guy. The authors of this chapter focus partly on winning strategies. One of their key results is that all the Flipping Coins positions are numbers. They also look very briefly at other related problems.

The second chapter is on the game of Blocking Pebbles, a two-player adaptation of Graph Pebbling. The blue-red-green version of Blocking Pebbles is shown to be PSPACE-hard.

The third chapter on *Transverse Wave*, an impartial color-propagation game inspired by social influence and quantum superimposition. The authors analyze the mathematical structures and computational complexity of the game.

The chapter *A note on numbers* considers the question "*When are all positions of a game numbers?*". The authors show that two properties known as F1 and F2 are necessary and sufficient. Some illustrative examples are given, and a warning is also included.

The chapter on *Ordinal sums, Clockwise Hackenbush, and Domino Shave* introduces and studies two rulesets. Clockwise Hackenbush is stated to be a new variant of Hackenbush Trees, while Domino Shave is the partizan version of Stirling Shave. The authors prove that the two are equivalent, despite the fact that Clockwise Hackenbush seems artificial.

Poset games are impartial combinatorial games whose game boards are partially ordered sets (posets). The chapter *Advances in finding ideal play on poset games* demonstrates systematic ways to compute the *nimber* (also called Grundy number) of poset games while permitting the categorization of winning or losing positions. The outcomes exhibit a comparison of ideal strategies on posets that are apparently unconnected. The authors focus on normal play games, where the first player having no move loses. They also study equality of games and some of its applications.

The chapter on *Strings-and-Coins and Nimstring are PSPACE-complete* obtains complexity-theoretic results for these two games. In fact, the authors show that Strings-and-Coins, a variant of the Dots-and-Boxes game (a simple pencil-and-paper game on a grid of dots) considered by Berlekamp, is indeed strongly PSPACE-complete on multi-graphs. This improves the NP-hardness result mentioned in the book *Winning Ways* by Berlekamp, Conway, and Guy. Finally, the authors pose some open problems.

Partizan subtraction games are two-player combinatorial games played on a heap of tokens. Each player is assigned a finite set of integers. A move consists in removing a number m of tokens from the heap, provided that m belongs to the set of the player. The first player unable to move loses. Fraenkel and Kotzig discussed these games, and they introduced the notion of dominance. In this chapter, the authors investigate other kinds of behaviors for the outcome sequence. In addition to dominance, three other disjoint behaviors are defined: weak dominance, fairness, and ultimate impartiality. The authors also study complexity and obtain several other results.

The chapter on *Circular Nim games CN(7, 4)* looks at Circular Nim. Circular Nim is a two-player impartial combinatorial game comprising n stacks of tokens set in a circle. A move consists of selecting k successive stacks and choosing at least one token from one or more of the stacks. The last player able to make a move wins. The question investigated by the authors is: *Who can win from a given position if both players play optimally?* The authors obtain results for $n = 7$ and $k = 4$.

Misère domineering on 2 × n boards studies Domineering, a tiling game in which one player places vertical dominoes, and a second puts horizontal dominoes, both flip-flopping turns until somebody cannot place on their turn. The authors state that past research has found game outcomes and values for some rectangular boards under normal play (last move wins); however, nothing was known about Domineering under misère play (last move loses). The authors look at misère outcomes of $2 \times n$ Domineering and the algebra of misère domineering. They study optimal-play outcomes for all $2 \times n$ boards under misère play. Misère outcomes for $m \times n$ Domineering are also shown.

The chapter on *Relator games on groups* specifies two impartial games, the Relator Achievement Game REL and the Relator Avoidance Game RAV. When presented a finite group G and a generating set S, both games commence with the empty word. Two players make a word in S by taking turns adding an element from $S \cup S^{-1}$. The first player to make a word equivalent in G to a former word wins the game REL but loses the game RAV. We can look upon REL and RAV as *make a cycle* and *avoid a cycle* games on the Cayley graph $\Gamma(G, S)$. The authors determine winning strategies for various families of finite groups including dihedral, dicyclic, and products of cyclic groups. Three player games and open questions are also discussed.

Playing Bynum's game cautiously examines a version of Bynum's game called Eatcake. Various sequences are used to analyze Eatcake. Two of these have terms with uptimal values. All others (eight) are determined by "uptimal+ forms," i.e., standard uptimals plus a fractional uptimal. The game itself is played on an $n \times m$ grid of unit squares. The author describes all submatrices of the 12×12 grid. Values of larger positions are also investigated.

Genetically modified games looks at the application of genetic programming to games. Genetic programming is often considered as a technique of evolving programs, starting from a population of unfit programs, fit for a particular job by applying operations similar to natural genetic processes to the population of programs. It uses crossover and mutation of genes representing functional operations. The authors introduce and solve two combinatorial games as well as present some advantages and disadvantages of using genetic programming. They study a combinatorial game whose ruleset and starting positions are helped by genetic structures. A single-point crossover and mutation game and a two-point crossover game are studied along with a crossover-mutation game.

Game values of arithmetic functions investigates two-player games inspired by standard arithmetic functions, such as Euclidian division, divisors, remainders, and relatively prime numbers. Games based on the aliquots, aliquants, and totatives, counting games, dividing games, factoring games, full set games and powerset games are discussed.

The next chapter is on a base-p Sprague–Grundy-type theorem for p-calm subtraction games . Base 2 arithmetic has a central function in combinatorial game theory. Nevertheless, a few games related to base p have also been found, where p is an integer greater than 1, although not necessarily a prime number. The author introduces a notion called as *p-calm* and points out that Nim and Welter's game are p-calm. Furthermore, using the p-calmness of Welter's game, the author extrapolates a connection between Welter's game and representations of symmetric groups to disjunctive sums of Welter's games and representations of generalized symmetric groups. Subtraction games and p-calm subtraction games are discussed.

The chapter *Recursive Comparison Tests for Dicot and Dead-ending Games Under Misère Play* applies the theory of absolute combinatorial games to formulate recursive comparison tests for the universes of dicots and dead-ending games. This is claimed by the authors as the first constructive test for comparability of dead-ending games under misère play using a novel category of end-games called *perfect murders*.

The chapter *Impartial Games with Entailing Moves* specifies a notion called *affine impartial*, which broadens typical impartial games. It analyzes their algebra by widening the conventional Sprague–Grundy theory with an ensuant minimum excluded rule. Results of Nimstring and Top Entails are given to exemplify the theory.

The next chapter is on Extended Sprague–Grundy theory for locally finite games and applications to random game-trees. The Sprague–Grundy theory for finite games without cycles was widened to general finite games in earlier works. The authors of this chapter state that the framework used to sort out finite games also extends to the case of locally finite games (that is, games where any position has only finitely many options). In particular, any locally finite game is equivalent to some finite game. The authors then examine cases where the directed graph of a game is selected randomly and is given by the tree of a Galton–Watson branching process. The authors study Extended Sprague–Grundy theory for games with infinite paths, reduced graphs, random game trees, and also provide examples.

The next chapter is on Grundy numbers of impartial three-dimensional chocolate-bar games. Chocolate-bar games are essentially versions of the Chomp game. A two-dimensional chocolate bar is a rectangular array of squares in which some squares are removed throughout the course of the game. A "poisoned" or "bitter" square, typically printed in black, is included in some part of the bar. A three-dimensional chocolate bar is essentially composed of a set of $1 \times 1 \times 1$ cubes with a "bitter" or "poison" cube at the bottom of the column at position (0, 0). Two players take turns to cut the bar along a plane horizontally or vertically along the grooves and eat the exposed pieces. The player who manages to leave the opponent with a single bitter cube is the winner. The authors determine Grundy numbers of impartial three-dimensional chocolate-bar games and also look at unsolved problems.

On The Structure of Misère Impartial Games studies the abstract structure of the monoid M of misère impartial game values. The author begins with the prerequisites and then demonstrates various new results, including a proof that the group of fractions of M is almost torsion-free. In addition, a method for estimating the number of distinct games born by day n, and some new results on the structure of prime games are illustrated.

3 Opinion

This book is indeed a special and well-prepared collection in honor of Elwyn Berlekamp, John H. Conway and Richard K. Guy, the mathematicians who contributed immensely to combinatorial game theory. The book is worth reading several times for those interested in game theory. It is a tribute to the work of the legends.