The Book Review Column[1]

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In this issue I want to spotlight two important new releases. Jeff Shallit's *The Logical Approach to Automatic Sequences* introduces the software Walnut for deciding first-order statements in Buchi arithmetic. This software has been used to prove hundreds of results in combinatorics on words. Can the junior's dream of replacing the pumping lemma with a similar software be far behind? *Data Science in Context* is a new textbook by Alfred Spector, Peter Norvig, Chris Wiggins and Jeannette Wing that addresses the breadth, oppportunities and perils of the field. Its new feature is a seven-element analysis rubric for determining data science's applicability to a proposed application.

Please considering reviewing these books or a new release that you find important and of interest to the TCS community.

1 This column

David J. Littleboy's review calls the textbook *Understand Mathematics, Understand Computing* by Arnold L. Rosenberg and Denis Trystram "a flawed gem." Although the concept, structure and material are all excellent, he finds issues with the book's informal, nonlinear presentation and overenthused style. The authors respond to the raised concerns and explain their pedagogical choices. I thank both the reviewer and the authors for sharing their viewpoints eloquently.

Reviewing a translated work is a challenge, because the reviewer must address the original content, the translation, as well as the commentary and explanatory matter accompanying the translation. James V. Rauff expertly discusses these aspects of *The Zeroth Book of Graph Theory: An Annotated Translation of Les Réseaux (ou Graphes) - André Sainte-Laguë* by Martin Golumbic, a monograph that captures graph theory research in its earliest days.

Leibniz on Binary by Lloyd Strickland and Harry Lewis is too an annotated translation of Gottfried Leibniz's writings on the binary system that insightfully traces the development of this important invention. In his review Bill Gasarch urges the reader to keep in mind the historical context when reading about Leibniz's exploration of possible applications of binary.

2 How to contribute

Please contact me to write a review! Either choose from the books listed on the next page, or propose your own. In either case, the publisher will send you a free copy of the book. Guidelines and a LaTeX template can be found at `https://algoplexity.com/~ntran`.

BOOKS THAT NEED REVIEWERS FOR THE SIGACT NEWS COLUMN

Algorithms & Complexity

1. Rubinstein, A. (2019). *Hardness of Approximation between P and NP* (ACM Books). Morgan & Claypool.
2. Knebl, H. (2020). *Algorithms and Data Structures: Foundations and Probabilistic Methods for Design and Analysis*. Springer.
3. Murlak, F., Niwiński D., & Rytter, W. (2023). *200 Problems on Languages, Automata, and Computation*. Cambridge University Press.

Data Science

1. Amaral Turkman, M., Paulino, C., & Müller, P. (2019). *Computational Bayesian Statistics: An Introduction* (Institute of Mathematical Statistics Textbooks). Cambridge University Press.
2. Nakajima, S., Watanabe, K., & Sugiyama, M. (2019). *Variational Bayesian Learning Theory*. Cambridge University Press.
3. Spector, A., Norvig, P., Wiggins, C., & Wing, J. (2022). *Data Science in Context: Foundations, Challenges, Opportunities*. Cambridge University Press.
4. Hrycej, T., Bermeitinger, B., Cetto, M., & Handschuh, S. (2023). *Mathematical Foundations of Data Science*. Springer.

Cryptography and Security

1. Tyagi, H., & Watanabe, S. (2023). *Information-Theoretic Security*. Cambridge University Press.

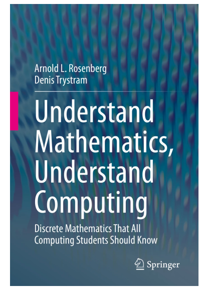
Combinatorics and Graph Theory

1. Beineke, L., Golumbic, M., & Wilson, R. (Eds.). (2021). *Topics in Algorithmic Graph Theory* (Encyclopedia of Mathematics and its Applications). Cambridge University Press.
2. Landman, B., Luca, F., Nathanson, M., Nešetřil, J., & Robertson, A. (Eds.). (2022). *Number Theory and Combinatorics: A Collection in Honor of the Mathematics of Ronald Graham*. De Gruyter.
3. Nowakowski, R., Landman, B., Luca, F., Nathanson, M., Nešetřil, J., & Robertson, A. (Eds.). (2022). *Combinatorial Game Theory: A Special Collection in Honor of Elwyn Berlekamp, John H. Conway and Richard K. Guy*. De Gruyter.
4. Shallit, J. (2022) *The Logical Approach to Automatic Sequences: Exploring Combinatorics on Words with Walnut*. Cambridge University Press.

Programming etc.

1. Sanders, P., Mehlhorn, K., Dietzfelbinger, M., & Dementiev, R. (2019). *Sequential and Parallel Algorithms and Data Structures: The Basic Toolbox*. Springer.

Review of [1]



Understand Mathematics, Understand Computing
Arnold L. Rosenberg and Denis Trystram
Springer, 2020
USD 73.61, Hardcover (also available in Softcover), 577 pages

Review by
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1 Overview

This book, subtitled *Discrete Mathematics That All Computing Students Should Know*, is, in addition to being a textbook for an introductory undergraduate course on discrete mathematics, an enthused, extensive, wide-ranging, and detailed overview of and introduction to the discrete mathematics that, well, all of us, not just computing students, really ought to know. My SB('76) and MS('84) in Comp. Sci. were from a distant past, and thus this book is exactly the review and overview I was looking for. However, while the book provided what I needed for the things that I missed or hadn't formally studied (in particular, SAT, graph theory, and statistics and probability), for the material that I was reasonably aware of, i.e., basic number theory, proof, logic, and sets, I found myself muttering "Huh? What are you talking about?" and running to other, more detailed, more technical, or more standard sources far more often than I should have had to.

Despite my kvetching (and most of this review will be kvetching), this book covers a lot of math in a compact package; it's the smallest and lightest of the competing textbooks at hand. It can be read on a favorite reading chair or sofa; the competing texts all require clearing space on a desk. Even the older ones from the 1990s.

Who is this book for? Although I have my doubts about this book as a textbook (it needs major editing of the writing style and careful rewriting to provide all the prerequisites and definitions a non-expert reader would need), it's definitely worth a read for someone, say, about to teach a course on this material or someone interested in reviewing the material with the intention of looking into it more deeply.

What's in the book? A true wealth of computation-relevant mathematics. The first two chapters are introductory material[2], followed by eleven chapters with subject-specific content[3] and seven similarly content-dense appendices. And the authors make a point of providing a proof (and often multiple proofs) when possible. Sure, the usual suspects (sets, logic, numbers, infinities, recurrence

[2] Chapter 1 is the usual bookkeeping for organizing a course, and Chapter 2 is an introduction to proof techniques.
[3] From sets and logic (Chapter 3), through several chapters on numbers and numbers systems, to chapters on combinatorics and graphs.

relations, counting and combinatorics, graphs) all make major appearances, but fun stuff, such as a proof of Fermat's little theorem, some of the joys of the Fibonacci numbers, and the clearest explanation of the Monty Hall problem I've ever read are here as well.

What I think this book is and what the authors think it is, differ. The authors' stated target audience is undergraduates from multiple fields who have not seen this material before. I'm not convinced. The main problem is that the authors often jump ahead of themselves into the content before providing the definitions and explanations needed. The authors' intention, perhaps, was to not write a boring, standard, ordinary textbook, and in avoiding that, they left out that which is good about ordinary textbooks: examples and orderly, thorough presentations. (Again, in reading this book, I found I needed to look for better definitions and descriptions of things in other textbooks far more often than I should have had to.)

But the authors disagree with me and argue strongly that their presentation of this material will enable students to actually do mathematics. Their claims are explicated in the "Manifesto" and Preface, which can be read in the preview on Amazon.

A word about the exercises. The exercises are more extensions of the text than the calculation exercises one would find in, say, a calculus text. While they are similar to those in other discrete mathematics texts, they extend the content of their chapters well. Solutions are provided for the most difficult exercises, and hints for the harder of the other exercises. There are not an excessive number of exercises, but they include interesting material. Problem 2 in Chapter 4 asks the reader to show how to compute the product of two complex numbers using only three multiplications *without any hints or solution*, thus indicating that the authors see it as an easy problem. (I remembered that there was a trick but was unable to remember or reinvent it, to my chagrin.) A problem in a later chapter revisits this idea (in the context of the multiplication of large integers) and does provide an answer. The Josephus problem is another such exercise.

2 How the Material is Presented

This book focuses, quite sensibly, on proofs. It also make a point of providing multiple proofs as often as possible, which is one of the things that attracted me to the book in the first place. However, "proof" here means algebraic proof from basic principles; no higher math. For a sophomore level textbook, this might seem a reasonable approach, but it has its problems. For starters, much of this material is number theory, yet number theory makes no appearance in either the table of contents nor in the index (although there are at least two occurrences in the text). My feeling here is that a short introduction to number theory would have made some of this material easier to present and understand. Similarly for abstract algebra: The book uses some fairly sophisticated concepts (e.g., free algebras and semigroups) with far from adequate explanation, if any.

In actual practice, this works for the majority of the material. But there are cases where this struck me as problematic, such as the discussion of Boolean algebras, which I mention below.

I noticed two proofs which were, for me, "incomplete". What I mean by that is that the last line prior to the QED was one algebraic manipulation short of getting back to the thing to be proved. For example, to show that the sum of the integers from 1 to m is $(m)(m+1)/2$, the last line in the proof as shown (from adding $(m+1)$ to both $1 \ldots m$ and the formula) is $(m+1)(m+2)/2$. But that needs one more step to get to $(m+1)((m+1)+1)/2$, i.e., the exact form of the formula being proved with $(m+1)$ replacing m.

This may sound like a quibble, but I don't think it is. In Michael Penn's YouTube videos [4], he is very careful to clearly show that he actually has gotten back to the thing to be proved. Maybe leaving out the final step as obvious is normal in published papers, but it's not the right thing for a textbook. A sophomore level undergraduate text needs to be more user-friendly.

Another issue which I think is important is that there are cases where the authors either don't define their notation before use or define it in the text in passing (i.e., not clearly marked as important), and that notation then becomes critical for a following discussion. One example of this is that $\mathbb{N}^+$ is used on p. 37 but is not defined until p. 106 and doesn't appear in the list of symbols at the back of the book.

Aside: In computer science, we find it natural to include zero in the natural numbers. So the authors' use of $\mathbb{N}$ and $\mathbb{N}^+$ to express the natural numbers including zero and the positive natural numbers is, of course, quite natural for computer science. But it is common to the point of being nearly universal in higher (that is, upper level undergraduate) mathematics texts to use $\mathbb{N}$ for the positive natural numbers. This needs to be explained, clearly and plainly, especially since this book claims to be aimed at students from multiple disciplines, not just computer science. (Both a recent number theory text [2] and a recent abstract algebra text [6] at hand use $\mathbb{N}$ for the positive natural numbers.)

I was not convinced by the informal introduction to proofs in Section 2.1. The basic story (as the authors tell it) is that before the 19th century, proofs were often deficient by modern standards. (Their example of a deficient proof is Fermat's proof of his last theorem. But since we don't have that proof, it's not an example of anything, let alone an example of a deficient proof.) The 19th century saw the development of modern, formal proof techniques. But these "ultimate-standard proofs made them quite unfriendly for humans to either craft or understand." My understanding is that this misstates the 19th century attempts at formalizing mathematics; that work was foundational work and was about assuring that proof was possible and that proofs were actually sound. I doubt that proofs of new theorems were ever required to use the Peano axioms. But nowadays, the authors argue, modern proofs are social exercises in which as long as everyone agrees it's a proof, then it's a proof. They give proof by induction as an example of a formalistic proof, and a proof using the pigeonhole principle as an example of a modern proof, saying "Are you convinced? If not, contact one (or both) of the authors, and we shall gladly provide more details." The pigeonhole principle example given seems to be a perfectly rigorous proof: all possible cases are enumerated.

I'm sympathetic to the author's desire "to overcome people's resistance to mathematical analysis and argumentation," but a "modern, human-friendly - but no less rigorous - methodology" strikes me as ultimately contradictory. There is a story to be told here, but telling it requires more care and thought.

Aside: there are at least two presentations of a proof of the infinitude of the primes (pp. 43-44 and pp. 237-238) that the authors seem to think is Euclid's proof but that differs from what other sources describe as Euclid's proof. Even worse, the authors incorrectly state "In fact, we claim n^* is a prime that is not in the sequence Prime Number." (Here n^* is 1 plus the supposedly finite product of all primes.) Uh, no. It's either prime or divisible by some number not in the claimed set.

Allow me to describe one of the problems I had with this book in detail. Chapter 3 has a section on "Boolean algebras." Having programmed in assembler on multiple architectures, I was

[4] https://www.youtube.com/c/MichaelPennMath

reasonably familiar with Boolean algebra. I thought. But even a rereading of the section left me confused. Consulting a variety of other texts (including Boole [1] himself) cleared up the problem. The authors had failed to explain that Boole developed (in the early 19th century) an algebra with radically different properties from ordinary algebra[5], and that that algebra turns out to be formally the same as the algebra of sets (which was developed in the late 19th century).

Since Chapter 3 is still introductory in nature, the authors chose not to introduce enough abstract algebra to describe this in its actual historical and logical structure, but rather chose to introduce set algebra first and then base the rest of the chapter on those ideas. Other textbooks describe the mathematics. In *Discrete Mathematics with Proof* [3] on p. 59 Gossett writes "A Boolean algebra, $\mathcal{B}$, consists of an associated set, B, together with three operators and four axioms." and then describes the components with examples. Gossett's example of using a power set as the carrier set makes the formal identity between Boole's algebra on truth values and Boolean algebras on sets blindingly clear. Getting this right takes less than two pages, including examples, if one has already presented the basics of abstract algebra.

Moving on from sets to propositional logic, the authors write "As we describe and define the basic connectives of the Propositional Logic, we point out their relationships to the Boolean set-related operations introduced in Section 3.2.2." So the bottom line is that the authors chose to present sets first, and operations on truth-valued variables as something related to those set operations, although that's logically and historically backwards. Allow me to quote the authors again: "Boole is generally credited with inventing these Boolean algebras." You, dear reader, may think that I'm overreacting here, but to me, this is simply wrong; Boole explicated the first of this class of algebras. He figured it out first, and gets his name on them because of that. The authors also write "... Boole is historically credited with developing the system we are describing here, with the goal of encapsulating a simple version of mathematical logic within an algebraic framework." I find these strangely insulting to Boole, who discovered the basic principles with which all modern digital computers operate.

In my first reading of Chapter 3, I thought I was still reading introductory material, and many of the things I see as problematic in Chapter 3 are a result of the authors planning to cover this material in more formal detail in later chapters. The presentation could function as a review of this material, but as either an introduction or as a textbook that expects the students to master the material, it strikes me as inadequate. Furthermore, Chapter 3 fails to explain its intellectual approach. Sets are, according to the chapter title, "The Stem Cells of Mathematics," yet the reader is left to infer for herself what that means. Ditto for "Structured Sets," which covers relations and functions (and some other things) described using set-theoretic notation. While I found it an interesting challenge to try to figure out what the authors had in mind, an undergraduate first encountering this material may be less amused.

Aside: The use of predicate logic in proofs is hard to explain. In everyday English, "implies" does a lot of work; in particular, it feels as though causality is being stipulated. But in predicate logic, it's just another binary operator on truth-valued variables. The authors, to their credit, discuss this. Several other introductions to the use of logic in proofs I've read failed to address this issue. So, kudos. This shows that the authors' idea that there is a need for the book this one tries to be is quite correct.

[5]Pinter [5] calls it "An even more bizarre kind of algebra".

3 What I Wrote in the Margins

The authors present a proof of Fermat's Little Theorem. This is a powerful tool that can obliterate seemingly impossible problems (such as determining the last 2 digits of 3^{400} [2], p. 55) in a flash. But the authors give no examples of its use. Thus the marginal notation "Examples, please!". On the same page I also wrote "Number of strings of length p over an alphabet of size $a = a^p$ and how that's equal to $a \bmod p$", which I thought was needed to understand the proof[6]. Which on the next page defines the "period" to be one less than the number needed to replicate the prior state (a shifted word in this case). The proof seems fine. It's just that using "period" to mean "one less than the what anyone else would see as the period" irritated. In a larger expression, the authors wrote $(i, i+1 \bmod n)$ and I wrote "an edge from node i to node $i+1$". My point here is that a lot of the work of reading this book lies in figuring out what the authors intended, and that they perhaps should have done more of that work. Maybe. The reader is allowed to think that some of this is my fault, not that of the book. But, I submit, if I need more help, undergraduate students will as well.

Another example: I scribbled "Doesn't define spanning tree." The section describes a spanning tree as being a graph with the same nodes and a subset of the edges, but fails to discuss what spanning means or why removing edges necessarily can create a tree. Other texts, e.g., [3], do. The things it does say about spanning trees are useful, interesting, and true. But they left out the definitions. Again, is this a quibble? I think not, because, overall, this book too often fails to provide definitions, either ahead of where they are needed by the reader, or at all. Thus the book comes across as inadequately user-friendly.

4 Quality of the Writing

The writing is enthused. The authors really like this material. Thus there are exclamation marks, superlatives, and metaphors throughout the text. In my opinion, this is excessive. My concern here is that while this writing is perfectly comprehensible to me, I worry that it could be found seriously irritating by the main target audience for this book: post-Millennial college undergraduates in the 2020s. Furthermore, many of them will be speakers of English as a second language. Words such as "betoken" will make an already linguistically difficult field that much more difficult. I'm not a fan of the term "user-friendly" but the lack thereof in this text is problematic.

There are two specific problems with the writing. The lesser of these problems is that the writing is outdated. The authors are fond of capitalizing things that shouldn't be capitalized. They write "the Theorem tells us that" when "this theorem tells us that" should be used. Terms such as propositional logic are capitalized, which, to the best I can tell, is no longer standard usage. There is also an overuse of double quotes. Almost every time something appears in double quotes, it means that the sentence in question needs to be rewritten. Randomly opening the text I found 'we now provide a "peek" into that area by means of...'. Inversely, the authors' use of italics is fine: it effectively brings out the point intended. The authors also seem overly fascinated by Latin, in at least one case using a Latin plural form. Again, this is not acceptable when your audience may include ESL speakers.

The more serious of the problems is that the writing is overly enthused, overly metaphorical,

[6] Yes, I realize that this sort of marginal note reflects my efforts at figuring out the proof, not necessarily a problem with the text.

and overly trite. Regarding infinitesimals, the authors write "The question of earliest discovery is one of the great real-life mysteries of all time." Uh, no. It isn't. On p. 91 the authors write: "In the lingo of the cognoscenti (the 'in-crowd'), these expressions are said to be in POS form, shorthand for (logical) product of (logical) sums." This particular example is, of course, embarrassingly bad in the extreme. Someone should have told the authors that.

5 Conclusions

There are a lot of things I like about this book, so I'm not happy that this sounds like such a negative review. It covers an amazing amount of mathematics in great detail. And that detail means that, if you work through the examples, you will have the preparation you will need for an upper level class, for example, one based on [4]. In addition, the later chapters, on combinatorics and graphs, provide an excellent introduction and preparation for further study. And even more challenging material is presented in the appendices. The bottom line, though, is that it's a flawed gem that needs work. In particular it needs two things. First, it needs an editor to bring the language up to ordinary, modern textbook standards and to reduce the rhetorical excesses. Second, it needs to be more generous with more detailed explanations of the technical terms, proofs, and derivations. As it is, I think that this book would be difficult for college sophomores. These, however, are low-level criticisms. The basic concept of the book, the overall structure of the book, and the material presented are all excellent: it deserves a better implementation.

References

[1] Boole, G., *The Complete Works of George Boole*, Shrine of Knowledge, 2020.

[2] Dujella, A., *Number Theory*, Školska knjiga, 2021.

[3] Gossett, E., *Discrete Mathematics with Proof*, Wiley, 2nd. ed., 2009.

[4] Graham, R. L., Knuth, D. E., Patashnik, O. *Concrete Mathematics*, Addison-Wesley, 2nd, ed., 1994.

[5] Pinter, C. C., *A Book of Abstract Algebra*, Dover, 2010.

[6] Weintraub, S. H., *Introduction to Abstract Algebra: Sets, Groups, Rings, and Fields*, World Scientific Publishing, 2022.

Comments on
Review by David J. Littleboy of our book
Understand Mathematics, Understand Computing

Arnold L. Rosenberg and Denis Trystram

We thank Mr. Littleboy for the substantial work of reviewing out 550-page book. We wish to comment on several aspects of his review. Our comments focus on broad issues relating to our book which we hope will provide the potential reader with helpful perspective.

Based on our long experience in teaching Discrete Maths both at the undergraduate and graduate level, we decided to write a textbook that does not correspond to writing in the historical sense. We do not aspire to teach Mathematics as a compendium of facts and tools but, rather, as a way of thinking and communicating and reasoning. We view readers as apprentices (initially) and collaborators (eventually). This approach gives our book the feel of a coherent tapestry of ideas which interconnect in often quite-nonlinear ways. The "forward" references that Mr. Littleboy mentions with discomfort intentionally suggest to the reader that what we call "doing" mathematics is a process wherein there is often more to say about a subject as we develop more background and intuition. We always strive to keep the reader apprised of the path we are following, via appropriate references and extensive discussion. In addition to being teachers, we play the role of guides as we develop the various mathematical topics our book contains. We help assimilate the readers (especially the most junior ones) by always providing readers several proofs of important results—indeed, several *types of proofs* built on a combination of text (for the textual thinker) and figures (for the non-textual thinker). We go far beyond the practice of other textbooks in these regards, as we help readers to enter the abstract world of mathematics.

1 Mathematics as a Way of Thinking

Mathematics is far more than a collection of facts generated by a set of concepts enhanced by tools for manipulating the concepts. Deep philosophical issues abound and connect us to the ontological foundations (i.e., true nature) of *mathematical "objects"* such as numbers, functions, relations (naming just a few)—and of the representational aspect of these objects—which *importantly* are what we compute with. A few examples will suffice:

- Exemplifying *mathematical "objects"*:
  - What is "nothing"? How does the concept *zero* represent "nothing"? Are there multiple candidates for a *zero*, which capture this concept in distinct ways?
  - At the other end of the spectrum, what is "infinity"? In what ways does inifinitude differ from finitude? Is there more than one valid — i.e., logically consistent — notion "infinity"?
- Exemplifying the *mechanisms that underlie mathematical reasoning*: Formalizing hypotheses, decomposing arguments into steps, invoking logical inference, and writing the proof that is our ultimate goal.

  - What is the essence of logical reasoning? of logical argumentation?
  - What does it mean to say that one proposition *implies* another?
  - When has one established that two propositions are "equivalent", in the sense that logical arguments cannot distinguish them?

The preceding are, of course, *foundational* questions, but each has *operational* analogues, as suggested by the following observations.

A crucial step in doing mathematics can be termed *modeling*: developing mechanisms for explicit reasoning, formalizing hypotheses, decomposing complex phenomena into simpler components, and making the logical inferences that ultimately lead one to mathematical proofs about real phenomena.

- Exemplifying the described processes and goals:

  Over the millennia, people have developed mathematical systems – collections of objects (sets, numbers, etc.), with relations that expose connections among objects, and with repertoires of operations that transform the objects. Much can be learned from studying such systems, including remarkable equivalences such as:

  - The following mathematical systems are "essentially equivalent", in a sense that can be formalized mathematically:
    * The system of *set algebras* – sets with operations such as union and intersection.
    * The system of *logical calculi* – logical formulae with operations such as conjunction and disjunction.
    * The system of *digital logical design* – circuit elements with operations such as **and** and **or**.

    Moreover, all of these systems form *Boolean algebras*.
  - The following computational problems are "essentially equivalent computationally," in the sense that an efficient solution-algorithm for any of these problems can be efficiently transformed into an efficient solution-algorithm for any of the others.
    * The problem of coloring the vertices of a given graph, using the fewest colors possible, in such a way that neighboring vertices have distinct colors.
    * The problem of scheduling meetings in the fewest individual rooms that guarantees privacy.
    * The problem of deciding whether a given graph can be drawn long a line in such a way that each pair of adjacent vertices are connected by an edge in the graph.

2 Mathematics within History and Culture

Systems such as Mathematics do not arise within a vacuum. We take pleasure in the book in discussing with the readers interesting historical/cultural aspects of Mathematics, even as we expose them to the technical aspects of the field. The aspects we allude to include linguistic and cultural origins of various mathematical concepts and terms. Examples include the Latinate and Greek origins of many terms and concepts, as well as the origin of the word "algorithm" and of historically named systems such as our Hindu-Arabic number system.

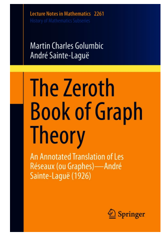
3 Explanation via Stylistic Convention

We have decided to honor the names of fields of inquiry via capitalization. This practice sometimes makes distinctions such as “Computer Engineering as a field” vs. “computer engineering as an activity” recognizable without distracting comments.

Similarly honoring the names of theorems, lemmas, etc., under discussion relieves the reader of the chore of decoding authors’ expositional intentions from their use of articles, such as “the” vs. “this”.

Review of [1]

**The Zeroth Book of Graph Theory:
An Annotated Translation of
Les Réseaux (ou Graphes) – André Sainte-Laguë
(1926)**
by Martin Charles Golumbic and André Sainte-Laguë



Springer Nature Switzerland, 2021
118 pages, Softcover, $54.99, eBook, $39.99

Review by
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1 Overview

The Zeroth Book of Graph Theory is the first and only English translation of André Sainte-Laguë's 1926 monograph *Les Réseaux (ou Graphes)*. Preceding the first full book on graph theory by a decade, Sainte-Laguë's monograph is a record of the state of graph theory research in its earliest days. The book begins with the basic definitions commonly known at the time and then moves on to a variety of graph-theoretic and combinatorial topics.

Martin Golumbic's translation maintains the illustrations, section numbers, and formatting of the original French version. It also preserves the mathematical notation used by Sainte-Laguë. The annotations include historical notes, modern equivalents of Sainte-Laguë's terminology, and corrections to mathematical and typographical errors. A glossary of French terms and their English translations chosen by Golumbic is provided along with a short biography of Sainte-Laguë and an extensive bibliography.

2 Summary of Contents

The following chapter summaries pertain to the chapters in *Les Réseaux (ou Graphes)*. I have also added a summary of the commentary provided by the translator/annotator to each chapter.

Chapter 1. Introduction and definitions. Sainte-Laguë motivates the monograph as a study addressing the topological problem of the possibility of certain relative placements of objects and the number of ways in which that placement can be made. He warns the reader against thinking that graph theory applies only to "curiosities" like the Königsberg Bridges and Four-Colour problems. The chapter proceeds with the standard definitions of beginning graph theory.

Golumbic's commentary situates Sainte-Laguë's work in its historical context. He reminds us that at this time most work in combinatorics was considered recreational mathematics.

Chapter 2. Trees. Sainte-Laguë investigates ways of characterizing trees and the use of their centers to count the number of non-isomorphic trees with n internal nodes.

Golumbic's commentary highlights Sainte-Laguë's use of pictures in his lectures at the Conservatoire National des Arts et Métiers.

Chapter 3. Chains and cycles. Sainte-Laguë discusses Eulerian cycles and paths and some combinatoric problems related to these. He also introduces Hamiltonian cycles. He cites Lucas' observation that a path through a labyrinth can be found by viewing the labyrinth as a graph and using depth-first search.

Golumbic's commentary addresses the Königsberg Bridges Problem and Lucas' work with mazes and recreational mathematics.

Chapter 4. Regular graphs. Sainte-Laguë studies a variety of topics related to regular graphs (all vertices have the same degree) with a focus on polygonal graphs. One question of interest is "Given a circle divided into equal parts, what is the number of distinct convex polygons with vertices among the division points?"

Golumbic's commentary presents the Petersen graph alluded to by Sainte-Laguë.

Chapter 5. Cubic graphs. Cubic graphs are regular graphs in which the vertices have degree three. In this chapter Sainte-Laguë investigates bipartite cubic graphs as a gateway to the study of the Four-Colour Problem.

Golumbic does not offer commentary in this chapter, but a footnote by Myriam Preissman situates Sainte-Laguë's work in the context of contemporary knowledge of the Four-Colour problem.

Chapter 6. Tableaux. Sainte-Laguë introduces incidence matrices and some graph properties obtainable from these matrices. His eye is on the result that "the edge-chromatic number (the minimal number of groups into which we can divide all edges so that two edges in the same group never have a common vertex) of a bipartite graph is equal to the maximum of its degrees."

Preissman provided the commentary for this chapter explaining Sainte-Laguë's argument and correcting errors/typos in the original text.

Chapter 7. Hamiltonian graphs. Sainte-Laguë associates permutations with regular Hamiltonian graphs and then classifies the permutations according to the properties of the corresponding graph. This leads to a discussion of the postage stamp problem: In how many ways can one fold a strip of postage stamps?

Golumbic's commentaries (1) suggest that Sainte-Laguë had seen the set of four stamps commemorating the 1924 Olympics and (2) point out the popularity of the mathematical analysis of fabrics, mosaics, and chessboards among 19th century mathematicians.

Chapter 8. Chessboard problems. Sainte-Laguë addresses the well-known problem of placing n chess pieces in non-attacking positions on a chessboard. His investigation includes rooks, queens, and knights. He also looks at variations of these problems, for example, placing n queens that attack exactly two others.

Golumbic's commentary provides a diagram for one of Sainte-Laguë's examples and an etymological remark on the French phrase *en prise* ("can be taken").

Chapter 9. Knight's tour. Sainte-Laguë looks at a variety of solutions to the Knight's Tour problem including those of Euler, Vandermonde, Bertrand, deMoivre, and Roget. The reader will find these different approaches a nice addition to the standard solution seen in modern texts. Sainte-Laguë finishes the chapter with a discussion of a relationship between the Knight's Tour solution and magic squares.

Golumbic's commentary informs us that the Roget mentioned by Sainte-Laguë is the same Peter Mark Roget of *Roget's Thesaurus* fame.

Chapter 10. Conclusion. This extremely short (two paragraphs) chapter sums up Sainte-Laguë's view that the study of graphs "can be pursued in many different ways, and in each of the notions defined may initiate new research" and "as limited as it may appear at first, is in fact vast and seems quite difficult." There is no commentary on this chapter.

The Zeroth Book of Graph Theory concludes with short biographies of André Sainte-Laguë and of Guy Ghidale Iliovici. (Iliovici was a friend and colleague of Sainte-Laguë's who lost his job in occupied France during WWII because he was Jewish. Iliovici was murdered by the Nazis at Auschwitz in 1942.)

The biographies are followed by a bibliography and a glossary of the French terms used by Sainte-Laguë and Golumbic's translations of them. The bibliography includes 223 works cited by Sainte-Laguë and an additional 80 provided by Golumbic. (I was surprised that no works by Frank Harary appeared in Golumbic's list.)

3 Opinion

Any assessment of *The Zeroth Book of Graph Theory* must be a tripartite discussion addressing the content of *Les Réseaux (ou Graphes)* itself, the English translation of the 1926 work, and the commentary and explanatory matter accompanying the translation.

a. *Les Réseaux (ou Graphes)* is an interesting look into the early development of graph theory. The reader will find many familiar themes in Sainte-Laguë's work. These include counting non-isomorphic graphs with specified properties, enumerating paths through graphs, and the popular chessboard problems. It was particularly interesting to see these early struggles with the Four-Color Problem.

   The style of presentation in 1926 was quite a bit different from what we are accustomed to in mathematical works. Theorems and proofs are not marked as such nor set off from the main text. Instead, results blend into the general narration. Any reader interested in the history of graph theory or looking for novel approaches to chessboard problems or the Knight's Tour will find items of interest in Sainte-Laguë's book.

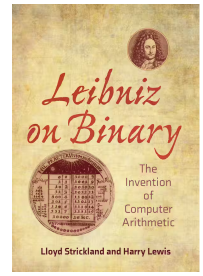
b. I cannot comment of the quality of Golumbic's translation because my command of French is not at the level of a translator. However, the glossary of French terms is quite extensive and would be useful for a scholar reading Sainte-Laguë in the original. I was able to follow along at a rudimentary level to get a feel for Sainte-Laguë's presentation and the choices Golumbic made in his translation. For those who read French, the original is available at `http://archive.numdam.org/item/MSM_1926__18__1_0.pdf`.

c. *The Zeroth Book of Graph Theory* is extensively footnoted. These include modern references as well as historical remarks provided by Robin Wilson, and technical remarks provided by Myriam Preissman and Alan Hertz. Corrections of mathematical and typographical errors were immensely helpful in reading the text. The footnotes include those of *Les Réseaux (ou Graphes)* as well as of *The Zeroth Book of Graph Theory.* When Sainte-Laguë has provided the footnote in the original monograph, Golumbic provides links to the work if it is currently available. This is an excellent feature because it allows the reader to see what Sainte-Laguë saw and inspired him to include it in his work.

   In general, the commentaries were informative. They connected the topics addressed by Sainte-Laguë to modern results, to historical context, and in some cases, to popular culture (e.g., *Star Wars*).

Although not a work from which one should begin one's study of graph theory, I can recommend *The Zeroth Book of Graph Theory* to anyone with an interest in the history of graph theory, the recreational aspects of graph theory, or the evolution of the style of mathematical exposition.

Review of[1]



Leibniz on Binary:
The invention of computer arithmetic
Authors of Book: Lloyd Strickland and Harry Lewis
Publisher: MIT Press
228 pages, Year: 2022
$35.00 Paperback

Reviewer: William Gasarch (gasarch@umd.edu)

1 Disclosure

I was a Harvard graduate student and Harry Lewis was my advisor, who is a co-author of the book under review.

2 Introduction

Whenever I read old papers in mathematics or computer science, I have the following thoughts:

a. Wow, these authors are smart! I think this because I know where their work leads and I read too much into what they did. So this is not quite correct.

b. Wow, these authors are stupid! I think this because I know where their work leads and I wonder how they missed the next "obvious" steps. So this is not quite correct.

Both of these thoughts are due to the arrogance of hindsight. We use Leibniz as an example.

a. Wow, Leibniz saw the potential for using binary for computation! He was smart! This viewpoint over-interprets what he did.

b. Wow, he thought that binary had analogies to Chinese and Christian thought. He was stupid. This viewpoint ignores the times he lived in.

This book gathers together Leibniz's writings on binary. Note that I said *writings* not *papers*. Indeed, these writings are mostly unpublished. Some are letters, some are just personal notes to himself. Since these were not meant for publication, some are rather unclear. This also adds another reason to avoid judging him by these papers: he did not intend them for the public.

The penmanship is awful, and it is sometimes hard to tell if a symbol is a typo, a stray mark, or something else. Hence it is great that the authors also summarize the writings.

The authors make the case that Leibniz really did invent binary arithmetic. While establishing that, they also discuss what Leibniz thought he could use it for. Spoiler alert: he thought it could be used for things we would never think of using it for now.

3 Summary of Contents

Since there are 32 short writings (a power of 2!) and they overlap, it would be madness to summarize each one of them. Instead I list some of his themes.

3.1 Computation

He often talks about how to express every number as 0's and 1's and how to do arithmetic. He compares it to base 10 by noting that in base 10 the numbers are shorter (which is good) but the computation at each digit takes longer (which is bad). He is (in modern terms) thinking about how fast an algorithm to add or multiply can be. He also writes about when adding many numbers together in base 2, if the carry is large make it a smaller carry but on a later column. This is clever and faster than the standard technique. Realize that he does not write things like *adding n m-bit numbers can be done in $O(nm)$ time*, nor should we expect him to. He also invented base 16 to help with computation.

Leibniz *designed* a calculating machine based on binary. He did not give much detail about his device. However, (1) it was hard to build and never was built, and (2) it would not have worked even if it had been built. This has a similarity and difference with Babbage's difference engine. Babbage's difference engine was never built, but it would have worked. Both Leibniz and Babbage were ahead of their time in that their mechanical devices are clumsy by comparison to later electronic devices.

3.2 The Power of Binary

He thinks base 2 will be very insightful. A direct quote:

> *... every secret of arithmetic and geometry lies hidden in this system.*

While this seems over-the-top, he did use base 2 in math:

a. In Chapter 4 he does use binary to understand perfect numbers. This is reasonable, since it was known that if $2^n - 1$ is prime then $2^{n-1}(2^n - 1)$ is perfect. However, while it is reasonable, I don't think it ends up being insightful.

b. Several times he writes the squares (or other sequences) in base 2 and notices that each column is periodic. He does not seem to prove this so much as delight in it.

The notion that base 2 leads to some insights in number theory is reasonable. The notion that base 2 leads to some insights in geometry is plausible. However, Leibniz also thinks that base 2 will lead to insights in theology. How? He says that 1 represents God and 0 represents nothing, and God made the world out of nothing. He also seems to say that there is an analogy between the following:

- Base 2 reveals an order and harmony that is not apparent in base 10.

- God's perspective reveals an order and harmony that is not apparent from our perspective.

Few 21st century people would find these thoughts interesting.

While this all seems silly, there are people today who claim that Quantum Computing will solve world hunger.

3.3 Chinese Hexagrams

In China there were some writings that used markings called *hexagrams*. There seemed to be 64 hexagrams. Leibniz claimed that the key to understanding these markings was base 2. Not only is this easily refuted now, it was then (and one of the letters to Leibniz did this). Even so, this notion appears in many of Leibniz's writings, and was repeated by others.

4 My Opinion

This is history done right. Too much math history is speculative and not based on much (e.g., Pythagoras had little to do with the theorem that bears his name, Gauss did not know $\sum_{i=1}^{100} i = 100 \times 101/2$ when he was 6 years old, Fig Newtons were not named after Issac Newton). The translations seem accurate, and the authors take great care in trying to figure out the order the writings were written in. More important is that we can really get insight into what Leibniz was thinking.

Doing history right sounds like a good idea. And it mostly is. But the drawback is that it is harder to distill it down to simple tell-able stories. And the 32 writings at times get repetitive. Even so, at the end of the book we find out both how smart Leibniz was and how stupid Leibniz was.

Leibniz deserves to have a cookie named after him. In fact, he does. See `https://en.wikipedia.org/wiki/Leibniz-Keks`.